TRANSFORMATION OF HUNGARIAN UNIFIED NATIONAL AND GAUSS-KRÜGER PROJECTION SYSTEM INTO WGS-84

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Abstract

A transformation between the two most commonly used Hungarian map projection WGS-84 ó Unified National, and WGS-84 ó Gauss-Krüger systems is presented. An algorithm and a computer program has been developed to carry out these conversions, and the accuracy is discussed.

INTRODUCTION

In the scope of *Austrian-Hungarian intergovernmental project 1995: "Application of satellite positioning and the development of data bases for GIS"* a scientific co-operation has been developed between the Institute for Applied Geodesy in the Technical University of Graz and the Department of Geodesy in the Technical University of Budapest.

The co-ordinates achieved by GPS are related to a global and geocentric frame. In order to relate these co-ordinates to a local (national) co-ordinate frame a transformation must be performed. One of the most important objective of the co-operation is the derivation of representative transformation parameter for both countries and the transformation between Austrian and Hungarian map projection systems.

The transformation between the two countries may be performed in two steps. First we should transform any kind of map projection systems to the WGS-84 ellipsoidal co-ordinates in one country, and then we should transform from the WGS-84 ellipsoidal co-ordinates to the desired system for the other country. In this study the first step is presented: the transformation between WGS-84 and the two most commonly used Hungarian (Unified National, and Gauss-Krüger) map projection systems.

1. TRANSFORMATION ALGORITHM

Conversions between map projection systems may take place either by the so called co-ordinate method (with closed mathematical expressions) or through transformation equations (polynomials), which were provided by using so called common points that have known co-ordinates in both systems.

It is possible to make exact conversions with closed mathematical expressions in cases only when both projection systems has the same reference surface and points of the same triangulation network coming from the same adjustment are represented in both projection systems (HAZAY 1964; VARGA 1981, 1982, 1986).

In each case when the conversion is not possible by closed mathematical formulas, the transformation can be performed only by transformation equations, which were deduced as polynomials from so called common points that have co-ordinates in both projection systems. In this case maximum five-order conformal polynomials can be applied depending on the number of common points (RULES FOR THE APPLICATION OF UNIFIED NATIONAL PROJECTION, 1975). For example, the connection between x, y co-ordinates of the projection system *I.* and x', y' co-ordinates of the projection system *II.* is established by the

$$\begin{aligned}
x' = {} & A_0 + A_1x + A_2y + A_3x^2 + A_4xy + A_5y^2 + A_6x^3 + A_7x^2y + A_8xy^2 + A_9y^3 + \\
& A_{10}x^4 + A_{11}x^3y + A_{12}x^2y^2 + A_{13}xy^3 + A_{14}y^4 + A_{15}x^5 + A_{16}x^4y + A_{17}x^3y^2 + \\
& A_{18}x^2y^3 + A_{19}xy^4 + A_{20}y^5 \\
\\
y' = {} & B_0 + B_1x + B_2y + B_3x^2 + B_4xy + B_5y^2 + B_6x^3 + B_7x^2y + B_8xy^2 + B_9y^3 + \\
& B_{10}x^4 + B_{11}x^3y + B_{12}x^2y^2 + B_{13}xy^3 + B_{14}y^4 + B_{15}x^5 + B_{16}x^4y + B_{17}x^3y^2 + \\
& B_{18}x^2y^3 + B_{19}xy^4 + B_{20}y^5
\end{aligned} \tag{1}$$

polynomials. Coefficients $A_0 - A_{20}$ and $B_0 - B_{20}$ (altogether 42 coefficients) can be determined by using common points suitably through an adjustment process. In such a case slightly different co-ordinates will be resulted after the conversion process depending on the position and number of selected common points and the applied method.

The conversion logic between the different map projection systems can be overviewed on *Fig. 1*. Transformation paths – and their directions – between different systems are pictured by arrows. It can be seen that it is possible to convert between both WGS-84 ↔ Unified National Projection (**WGSóEOV**) and WGS-84 ↔ Gauss-Krüger (**WGSóGAK**) systems only through other intermediate systems. E.g. if a conversion between **WGS** and **EOV** systems is needed then WGS-84 co-ordinates first have to be converted into the new Gaussian sphere (NGS) and then into an auxiliary plain co-ordinate system (SVR) and finally they should be converted from this SVR system into EOV co-ordinates; or e.g. if a conversion between **GAK** and **WGS** systems is needed then Gauss-Krüger co-ordinates first has to be converted into an auxiliary plain co-ordinate system (SVR) and then into the new Gaussian sphere (NGS) and finally they should be converted from the new Gaussian sphere into the WGS-84 ellipsoid.

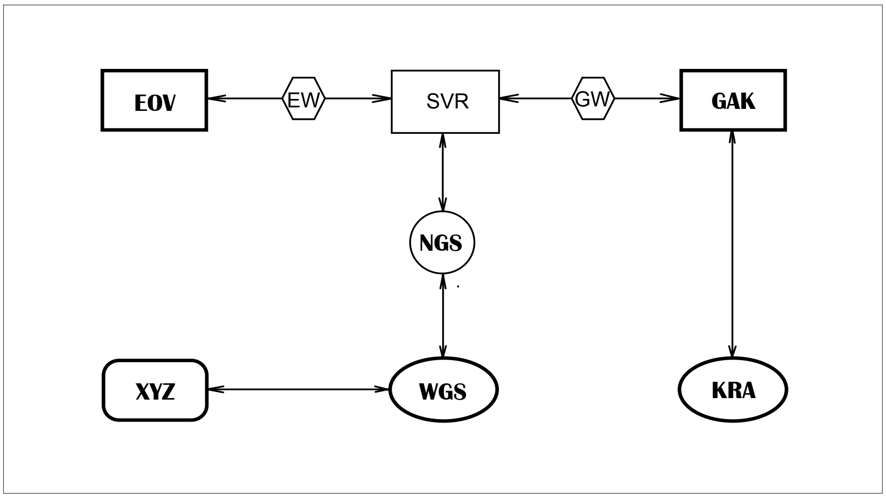

Fig. 1

If any two systems in *Fig. 1* are connected through a hexagonal block then between these two systems only an approximately accurate conversion could be made by the (1) transformation polynomials. In *Fig.1* the two-letter abbreviations in hexagonal blocks show which binary data files, containing transformation polynomials, have to be used to convert between the two neighbouring systems (their meaning is: EW contains the coefficients of transformation polynomials $A_0 - A_{20}$ and $B_0 - B_{20}$ in equations (1) for (**WGS**ó**EOV**) transformation, and GW contains the coefficients of transformation polynomials for **WGS**ó**GAK**) transformation.

If any two systems in *Fig. 1* are connected by a continuous line then an exact conversion by the co-ordinate method, i.e. through closed mathematical expressions can be made. The datum surface of Hungarian Gauss-Krüger system is the Krasowsky ellipsoid, and the transformation between Krasowsky and Gauss-Krüger system (**GAK**ó**KRA**) was prepared using the equations of Plewako (PLEWAKO 1991). There is an exact and very simple conversion between WGS-84 and the spatial Cartesian geocentric system (**WGS**ó**XYZ**) too – but in most cases we need not to use this transformation, because GPS serve both WGS-84 and the spatial Cartesian geocentric co-ordinates at the same time.

2. COMPUTER SOFTWARE DEVELOPMENT

We worked out a computer program package by which conversions can be made between map projection systems which can be seen on *Fig. 1*. Our software has two main parts: a module which yields coefficients of transformation polynomials and another module which performs actual conversions.

Programmes in *module 1* computes coefficients of transformation polynomials in equation (1). The main program of this module makes it possible to calculate the coefficients of polynomials when some common points are adequately given. This program creates binary data files containing coefficients of transformation polynomials, which are required for conversions between EOV – WGS-84 and Gauss-Krüger – WGS-84, and determines the degree of transformation polynomials automatically as a function of the number of common points. If there are *21* or more common points then all the (namely 42) coefficients of a five-degree polynomial in the expression (1) can be determined. When the number of common points lies between *15* and *20* then the degree of polynomials is *4*, if the number of common points is between *10* and *14* then the degree is *3*, and then the number of common points is between *6* and *9* the degree of polynomials required for transformation is *2*. At least *6* common points are necessary to compute coefficients of the polynomials, however an effort should be made to use as many common points as possible to determine these polynomial coefficients. If the number n of common points is such as $7 \le n \le 9$, $11 \le n \le 14$, $16 \le n \le 20$ or $n \ge 21$, than the number of equations is greater than it is necessary (the problem is over determined), hence the most reliable values of unknown polynomial coefficients are determined through an adjustment program.

Actual conversions can be made by Module 2. Two important programs can be found in this module: input-output organiser program of the conversion software, and the main conversion program. Co-ordinates of points to be converted can be inputted from both keyboard and disk files, a built-in special editor helps to handle co-ordinates from the keyboard or to transfer them into a work file in the required format. This special editor also serves to check inputted co-ordinates on a high level and therefore it is practically impossible to read erroneous co-ordinates. Co-ordinates from disk files will also pass through the above strict trouble shooting process and they will be transferred into a work file as well. Co-ordinates in this work file are transformed into the required system by the main conversion program.

It should be remarked another computer program had been developed earlier to carry out the transformation of remainder Hungarian map projection systems to the Unified National Projection (**EOV**) system (VÖLGYESI at all, 1994, 1996).

3. ACCURACY OF TRANSFORMATIONS

A conclusion could have been drawn as a result of our test computations that the accuracy of computed plane co-ordinates is *1 mm* and of geodetic co-ordinates is *0.0001″* if it is possible to convert through closed mathematical expressions between certain map projection systems. Such conversions are between (**GAK**ó**KRA**) and (**WGS**ó**XYZ**) systems.

In all other cases when the transformation path between any two systems passes through an hexagonal block in *Fig. 1*, the accuracy of transformed co-ordinates depends, on one side, how accurately the control networks of these systems fit into each other; and on the other side, how successful was the determination of transformation polynomial coefficients. It follows also from these facts that no matter how accurately these transformation polynomial coefficients was determined, if the triangulation networks of these two systems do not fit into each other accurately – since there were measurement, adjustment and other errors during their establishment – then certainly no conversion of unlimited accuracy can be performed (in other terms, only such an accurate conversion between two map projection systems is possible that the accuracy allowed by the determination errors or discrepancies of these control networks). This fact, of course does not mean that ones should not be very careful when the method of transformation is selected or – when the polynomial method is applied – the coefficients in Equation (1) are determined.

When the polynomial method is chosen the most important question is to determine the optimal degree of the polynomial. By considering a simple way of reasoning one could arrive at the conclusion that the higher the degree of the polynomial the higher the accuracy of map projection conversions will be. On the contrary, it could be proved by our tests that the maximum accuracy was resulted by applying five degree polynomials. No matter whether the degree was decreased or increased, the accuracy of transformed co-ordinates was lessened alike (more considerably by decreasing, less considerably by increasing).

It is true, really, that minimum 21 common points are required to determine coefficients of a five degree polynomial, but our experiences revealed the fact that the accuracy of conversions can be increased further on by using a considerably greater amount of common points and the most probable values of these unknown polynomial coefficients are determined through an adjustment.

The program which calculates the coefficients of polynomials, provides information characteristic to the accuracy of conversions made by polynomial method. Coefficients of transformation polynomials are first provided by the program based on co-ordinates of common points y_i', x_i' and y_i'', x_i'' in systems I and II, respectively. Then y_i', x_i' co-ordinates in system I are transformed into co-ordinates ty_i'', tx_i'' in system II by using these coefficients and finally the standard error characteristic to conversion,

$$\mu = \sqrt{\frac{\sum_{i=1}^{n}(ty_i''-y_i'')^2 + \sum_{i=1}^{n}(tx_i''-x_i'')^2}{n}} \qquad (2)$$

will be determined.

It could be mentioned that for example, between WGS-84 and EOV systems the standard error is ±50 mm from the expression (2) for the complete area of Hungary when 43 common points are used; and there is ±62 mm standard error between WGS-84 and Gauss-Krüger systems by using 34 common points respectively.

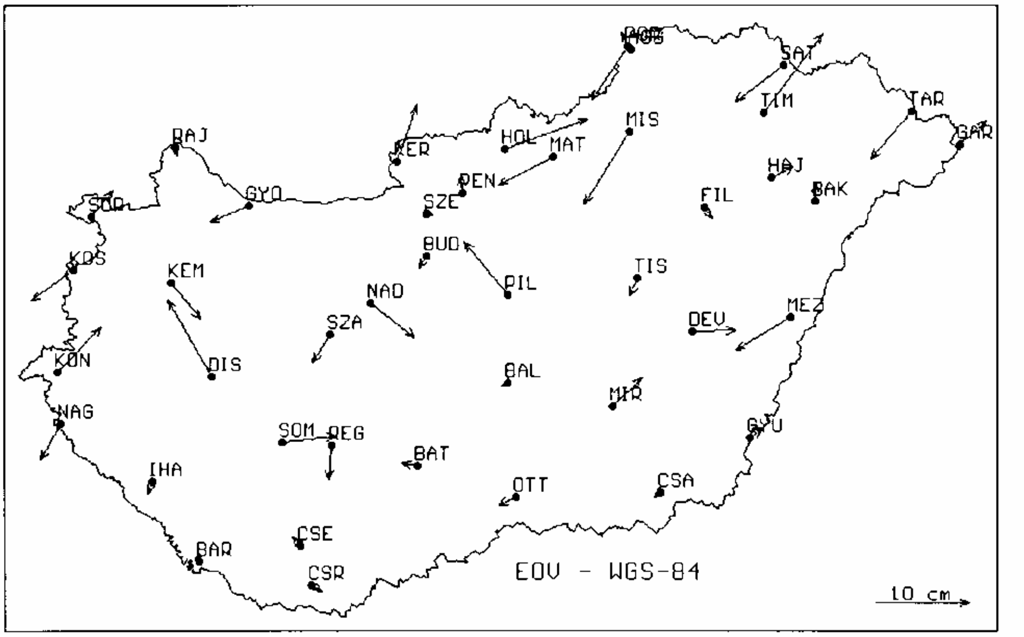

Fig. 2

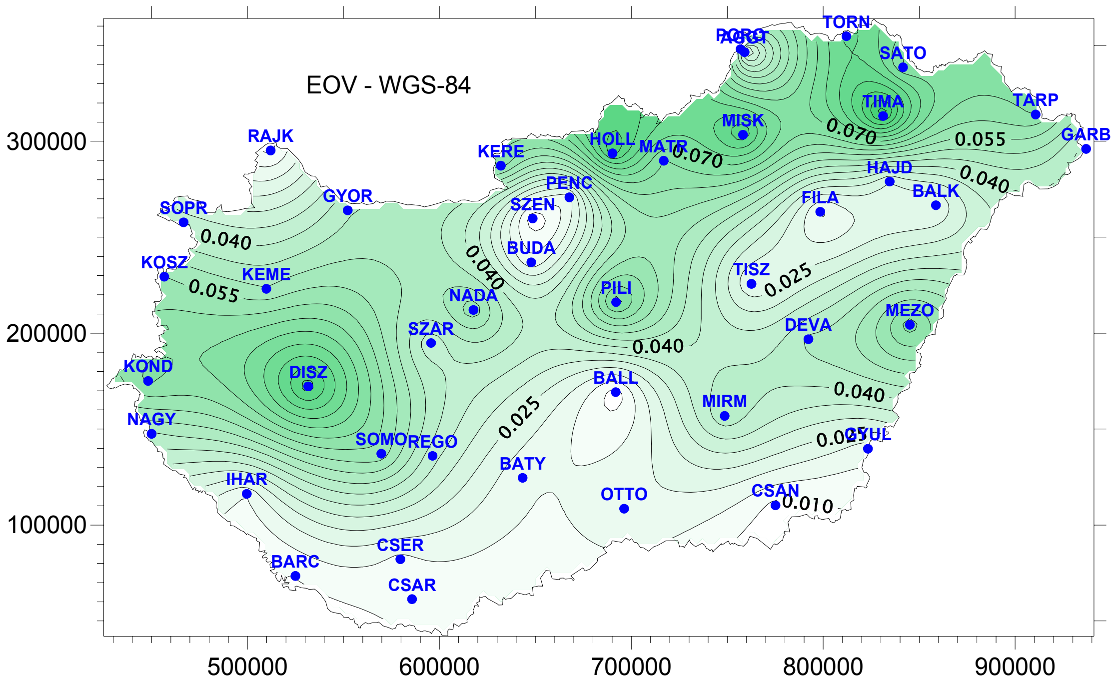

Fig. 3

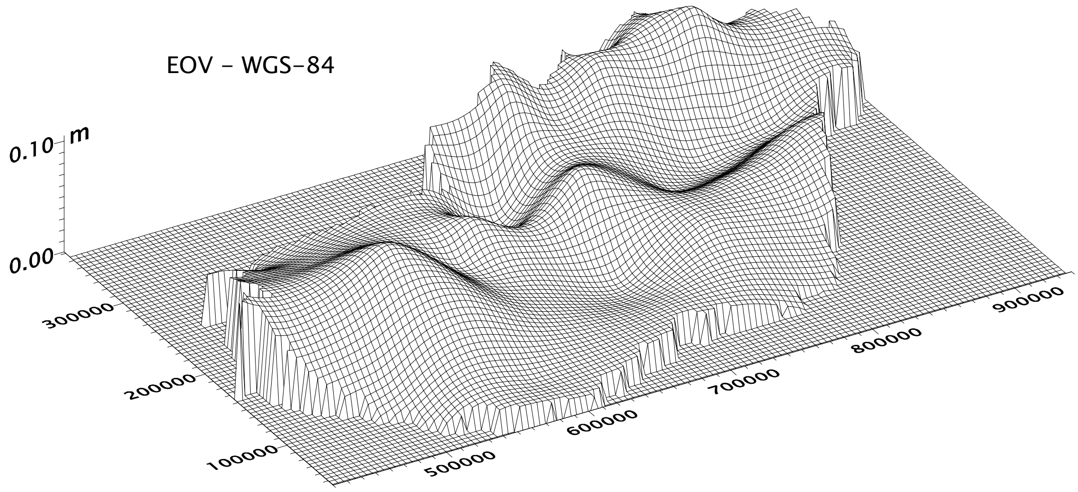

Fig. 4

Figs. 2, 3, 4 show the distribution of errors of transformation between WGS-84 and EOV systems. A vectorial picture of errors was drawn in *Fig. 2*. The x axes of co-ordinate system points to *North* (upward in *Fig. 2*) and y points to *East* (to right in figure), and the length of the vectors are

$$\sqrt{(ty_i''-y_i'')^2+(tx_i''-x_i'')^2}\,. \tag{3}$$

The same errors are visualised on an isoline map in *Fig. 3* and on a surface map in *Fig. 4* at the same time. It can be seen from the pictures, there is no place where the errors are greater than *10 cm* anywhere in Hungary. The biggest errors occur at the North-eastern part and the middle-western part of the country, and there are big continuous areas where the errors are only a few centimeters.

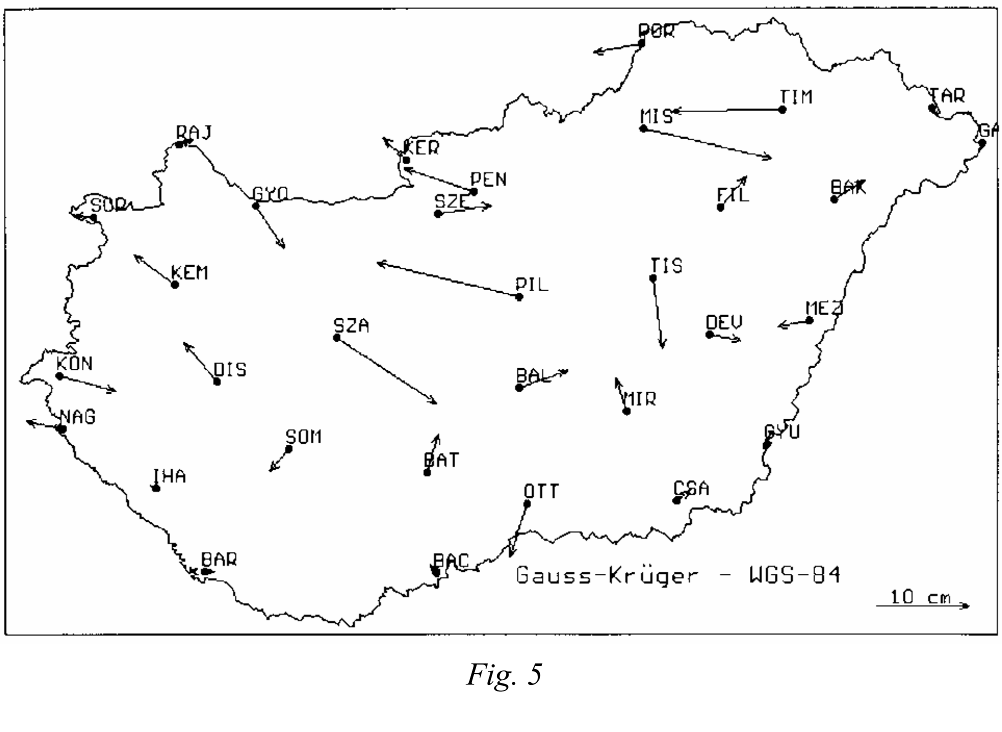

Fig. 5

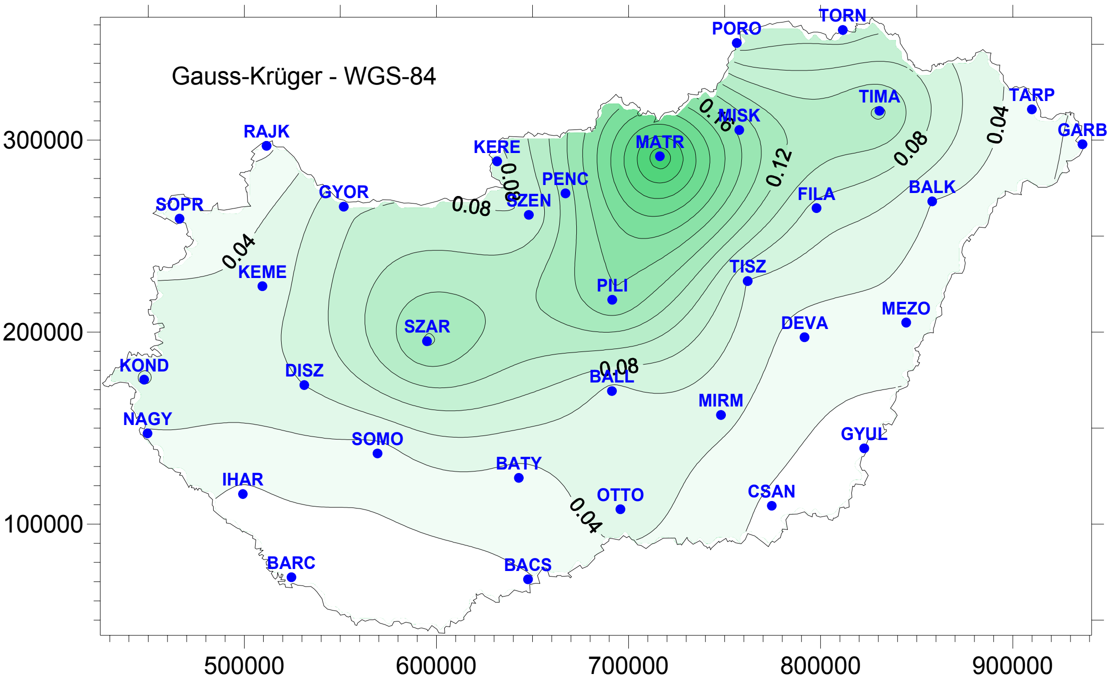

Fig. 6

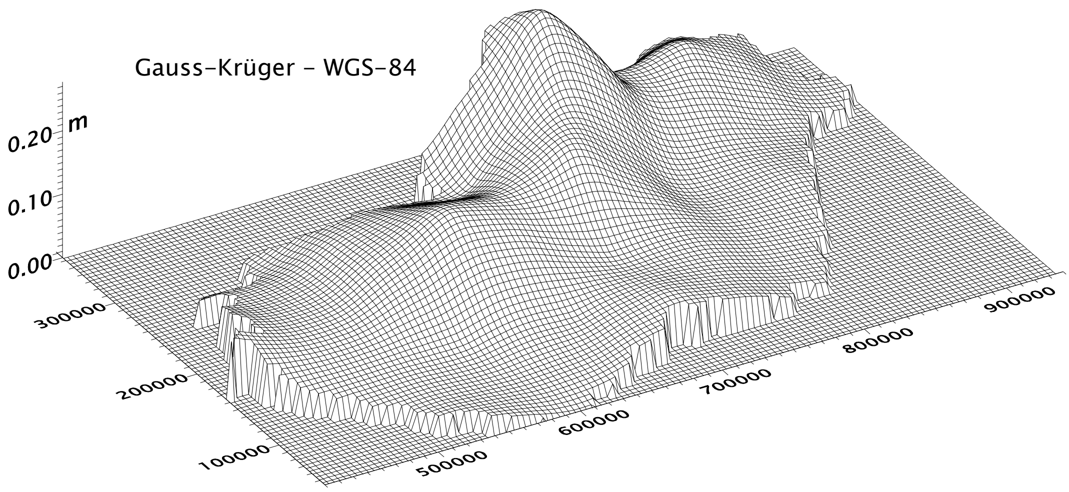

Fig. 7

Figs. 5, 6, 7 show the distribution of errors of transformation between WGS-84 and Gauss-Krüger systems. A vectorial picture of errors was drawn in *Fig. 5*. The x axes of co-ordinate system points to *North* (upward in *Fig. 5*) and y points to *East* (to right in figure), and the length of the vectors was computed by the equation (3). The errors are visualised on an isoline map in *Fig. 6* and on a surface map in *Fig. 7* too. The maximum error is *14 cm* in the country. The biggest errors occur at the North-eastern and the middle part of the country, and there are big continuous areas where the errors are only a few centimetres, similarly to the EOV - WGS-84 transformation.

Our experiences showed the fact that although the accuracy can somewhat be increased by increasing the number of common points within the polynomial method but the accuracy of conversion can not be increased beyond a certain limit even with this method since there is a difference between the two triangulation networks. In certain cases, however, an improvement could be gained when transformation polynomial coefficients are not determined for the complete area of the country but for only smaller sub-areas common points are given and transformation polynomial coefficient are determined.

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