Determination and reliability estimation of vertical gradients based on test measurements

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Abstract. Gravity values should be referred to benchmarks, but absolute and relative measurements give *g* value at the reference height of instruments. The *vertical gradients* (VG) can be used for the reduction of measured gravity from the reference height of an instrument to the benchmark. In case of absolute or high accurate relative measurements, a very precise height reduction with ±1μGal accuracy is necessary, and using the normal value of VG (0.3086 mGal/m) is not sufficient for this purpose, because the differences between the normal and the real values of VG may amount to 20-25%. Values of the real VG can generally be determined by measuring gravity at different heights. Applying three gravimeters and measuring 3-4 series at four different heights the necessary accuracy (±1μGal) of height reduction can be reached, but it is expedient to determine and take into account the periodic errors of gravimeters by 1 mGal at the processing of measurements. Based on our experiences the best height reduction values come from the measurements by three gravimeters, measuring 3-4 series at four different heights, and applying the quadratic approximation for processing the measurements.

Keywords. Gravity, vertical gradient of gravity, height reduction of gravity

1 Investigation of vertical gradient by measuring gravity at different heights

In our earlier investigations it turned out that using the normal value (0.3086 mGal/m) of VG for converting the measured gravity from the reference height of an instrument to the bench mark does not provide sufficient accuracy, because the differences between the normal and the real vertical gradients may amount to ±20-25% in Hungary (CSAPÓ, PAPP 2000). Using the normal or the real vertical gradient (VG) for this conversion may cause a difference of the height correction of a measurement about 6 μGal. The purpose of our investigation is to find such a ratio of *g/h* , which makes a better reduction of gravity measured by absolute or relative gravimeters from their reference heights to the benchmark.

In our work the characteristics of variation of gravity with height is investigated rather than the variation of *g* referring to 1 m height in the adjustment. It is possible to investigate the nonlinearity of the ratio *g/h* too by measuring gravity at more than two different heights, and the height reduction may be computed directly from this function of g/h.

2 Mathematical model of the solution

We assumed that the corrected relative values of gravity measured by LCR gravimeters at different heights above a benchmark are available for the computation of VG. First, if the variation of gravity as a function of height is postulated to be linear and the observation equation is

$$\Delta g_i = \underbrace{\partial g / \partial h}_{VG} \cdot \Delta h_i \quad (i = 1, 2, \dots n\text{-}1) \tag{1}$$

where $\Delta g_i = g(h_{i+1}) - g(h_i)$, $\Delta h_i = h_{i+1} - h_i$; $g(h_i)$ are the measured gravity at different heights h_i (h_i are sensor heights of instruments above the benchmark) and VG is the unknown parameter in adjustment. Then, if the variation of gravity is postulated to be a quadratic function of height, the observation equation is

$$g(h_i) = g_0 + \underbrace{\partial g / \partial h}_{a} \cdot h_i + \underbrace{\partial^2 g / \partial h^2}_{b} \cdot h_i^2 \tag{2}$$

$$(i = 1, 2, \dots n)$$

where $g(h_i)$ are the measured gravity at different heights h_i and g_0, *a, b* are the unknown parameters in adjustment (g_0 is a fictitious mean value of initial level of gravity readings - an auxiliary parameter in adjustment). The adjustment was carried out by matrix-orthogonalization method (VÖLGYESI 1979, 1980, 2001) and software was developed for

VG computation under the Windows operating system.

3 Measurements at two different heights

Eight series were measured by three different LCR gravimeters at heights 50 and 1300 mm above the benchmark. The benchmark was placed on a pier at the ground level, and height differences were measured between the benchmark and the estimated sensor height. All LCR gravimeters were calibrated at calibration line. Periodic errors of gravimeters by 1 mGal at the processing of measurements were taken into account, and gravimeters were equipped by electronic levels. The computation results are summarized in Table 1 and 2. Height reductions δg are computed by $\delta g = VG \cdot h$ for the height $h=1m$ and VG is determined based on (1) by adjustment. In Table 1 height reductions and their standard deviations referring separately to each gravimeters can be found in *mGal* (1 mGal = 1000μGal = $10^{-5}ms^{-2}$). In Table 2, height reductions and their standard deviations referring to all LCR gravimeters can be found. On the right side of Table 2 the running mean values can be found (the mean values of the 1st and the 2nd rows in the 2nd row; the mean values of the1st, 2nd and the 3rd rows in the 3rd row; etc.).

Table 1. Height reductions and their standard deviation *m* for each LCR gravimeter based on measurements at two different heights, computed by $\delta g = VG\ h$ for the height *h=1m* (values are in [mGal])

	LCR-1919		LCR-963		LCR-821	
series	δg	m	δg	m	δg	m
1	-0.2509	0.0004	-0.2528	0.0035	-0.2525	0.0022
2	-0.2480	0.0011	-0.2515	0.0017	-0.2563	0.0080
3	-0.2510	0.0007	-0.2567	0.0061	-0.2513	0.0005
4	-0.2516	0.0012	-0.2527	0.0014	-0.2504	0.0042
5	-0.2507	0.0024	-0.2492	0.0015	-0.2522	0.0040
6	-0.2485	0.0022	-0.2575	0.0026	-0.2515	0.0020
7	-0.2523	0.0010	-0.2524	0.0067	-0.2504	0.0040
8	-0.2488	0.0025	-0.2536	0.0050	-0.2466	0.0005
mean:	-0.2502	0.0025	-0.2533	0.0050	-0.2514	0.0051

The standard deviations computed from measured series vary randomly between 0.4 and 8.0 μGal, and the mean standard deviations of the eight series vary between 2.5 and 5.1 μGal as it can be seen from Table 1. The biggest difference between the mean height reductions of gravimeters is 6.7 μGal. The high quality of measurements are demonstrated by the fact, that the biggest difference in the value of height reductions computed by the measurements of any one gravimeter is less then 10 μGal.

Table 2. Height reductions and their standard deviations for all LCR gravimeters based on measurements at two different heights, computed by $\delta g = VG\ h$ for the height *h=1m* (values are in [mGal])

	values by series		running means	
series	δg	m	δg	m
1	–0.2521	±0.0027	–0.2521	±0.0027
2	–0.2519	±0.0064	–0.2520	±0.0049
3	–0.2530	±0.0049	–0.2523	±0.0050
4	–0.2516	±0.0029	–0.2522	±0.0046
5	–0.2507	±0.0032	–0.2519	±0.0044
6	–0.2485	±0.0052	–0.2520	±0.0045
7	–0.2517	±0.0048	–0.2519	±0.0046
8	–0.2497	±0.0042	–0.2516	±0.0046

Running mean values on the right side of Table 2 show, that height correction is not influenced significantly by increasing the number of measurements (in our case the change is only 0.5 μGal after the eighth measurement with respect to the first value). This finding is supported by the fact that our earlier VG value (-0.2519) is very close to the value in Table 2 (-0.2516). Our earlier value of VG comes from 47 measured series by several gravimeters in the last years.

4 Measurements at more than two different heights

Investigation of the nonlinearity of VG is possible only in case of measuring the gravity at more than two different heights (CSAPÓ 1976, 1987, 1997). Our investigations about the nonlinearity of VG are based on test measurements at three and four different heights above the benchmark of the absolute gravity station in Budapest. Heights of measurements were set so that the sensing-mass of instruments would be at special heights above the benchmark (A = 206 mm, B = 911 mm and C = 1631 mm). Nine series were measured by each gravimeter with a sequence A-B-C-A-B-C-A-B-C-A.
The ratio *g/h* is determined by adjustment with both linear and quadratic approximation using Eqs. (1) and (2). Height reduction δg was computed in both linear and quadratic cases at $h=1$ m height. The equation

$$\delta g = VG \cdot h$$

is used in linear and

$$\delta g = \partial g / \partial h \cdot h + \partial^2 g / \partial h^2 \cdot h^2$$

in quadratic approximation to compute height reductions referring to each one and to the complete group of gravimeters.
In Table 3 the variation of height reductions can be seen as a function of repetition number of measurements n . Standard deviations indicate that increasing the repetition number of measurements gives an improvement of reliability of height reduction in quadratic approximation, but does not influence the differences of δg between linear and quadratic cases.

Table 3. Variation of height reductions as a function of repetition number of measurements

	linear		quadratic	
n	δg	m	δg	m
1	–0.2487	±0.0089	–0.2619	±0.0067
2	–0.2485	±0.0090	–0.2615	±0.0062
3	–0.2480	±0.0096	–0.2617	±0.0059
4	–0.2477	±0.0096	–0.2619	±0.0060
5	–0.2477	±0.0097	–0.2625	±0.0060
6	–0.2476	±0.0094	–0.2620	±0.0056
7	–0.2476	±0.0090	–0.2615	±0.0053
8	–0.2477	±0.0089	–0.2612	±0.0052
9	–0.2483	±0.0088	–0.2613	±0.0052

Height reductions were computed applying both linear and quadratic approximations for different heights based on all the measurements (27 measured series) by the three LCR gravimeters. These results and the differences between the linear and quadratic values are summarized in Table 4, and the linear and quadratic functions of height reduction can be seen in Fig. 1.

Table 4. Height reduction differences between linear and quadratic approximations

h [mm]	δg (linear)	δg (quadratic)	diff. [mGal]
206	–0.0512	–0.0563	0.0051
300	–0.0745	–0.0816	0.0071
500	–0.1242	–0.1343	0.0101
700	–0.1738	–0.1861	0.0123
900	–0.2235	–0.2365	0.0130
911	–0.2262	–0.2393	0.0131
1000	–0.2483	–0.2613	0.0130
1300	–0.3228	–0.3338	0.0110
1500	–0.3727	–0.3806	0.0079
1631	–0.4050	–0.4106	0.0056

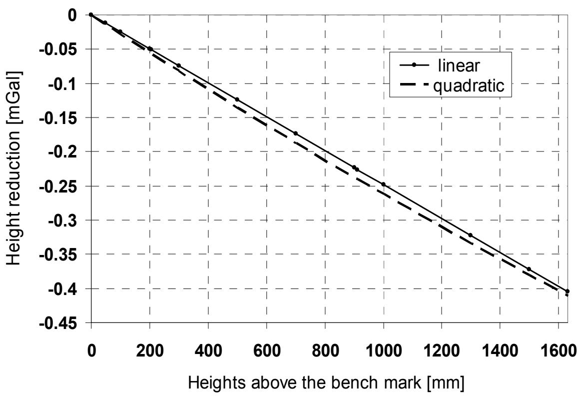

Fig. 1. Linear and quadratic functions of height reduction of gravity

In Fig. 2 the VG values can be seen, which were computed by Eq. (1) from measurements at three different heights (A = 206 mm, B = 911 mm and C = 1631 mm) above the benchmark at the absolute gravity station in Budapest. Dashed lines indicate VG values computed from measurements by each gravimeters separately, continuous lines indicate mean values of VG as the function of repetition number of measurements, which refer to each gravimeter separately, while the thick line indicates the mean value referring to all the gravimeters. It can be seen that the variation of mean value for all gravimeters is only 1-2 μGal.

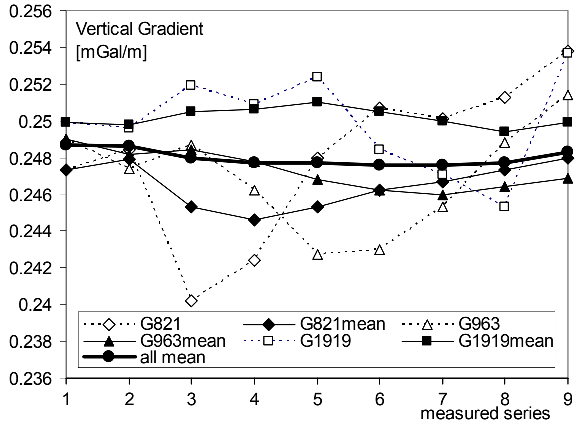

Fig. 2 VG values by linear approximation from measurements at three different heights

At the same time, it is necessary to recognize that measuring a large number of series by a single gravimeter does not give significant improvement of the VG's value.

It can be seen on Fig.1 and Table 4 that the differences of height reductions between linear and quadratic approximations are approximately 13 μGal at the middle of our height interval. This is very important in the case of absolute gravity measurement's reduction to the benchmark, because the absolute gravimeter's reference heights are mostly in the interval between *0.8-1.0 m*.

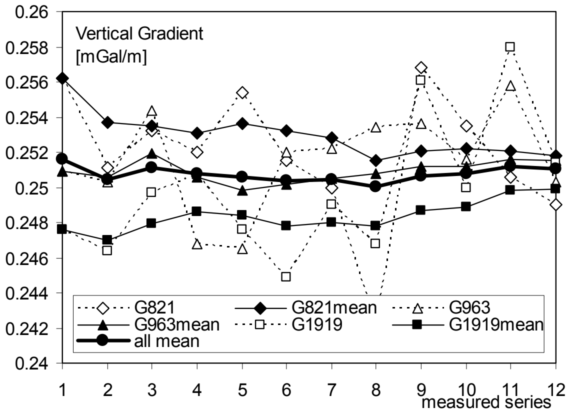

Fig. 3 VG values and running mean height reductions referring to each gravimeter by linear approximation from measurements at four different heights

On Fig. 3, the height reduction values and running mean height reductions computed to *1 m* height by linear approximation can be seen; measurements were taken at four different heights by three LCR gravimeters in 12 series with a sequence A-B-C-D-A-B-C-D-A-B-C-D-A. The nearly horizontal all mean VG line indicates that the mean value of VG do not vary significantly if we increase the number of measured series, so the height reduction can already be determined with sufficient accuracy applying only 3-4 series of measurement.

Table 5/a. Variation of height reductions as the function of repetition number of measurements at h = 206, 560, 911, 1631 mm heights

	linear		quadratic	
n	δg	m	δg	m
1	–0.2453	±0.0084	–0.2559	±0.0046
2	–0.2453	±0.0080	–0.2545	±0.0047
3	–0.2456	±0.0071	–0.2535	±0.0045
4	–0.2463	±0.0072	–0.2532	±0.0048

In order to provide height reduction with ±1 μGal accuracy, we set additional heights for test measurements, and further investigations were made to determine the optimal repetition number of measurements at four different heights using three gravimeters. The relevant data can be found in Table 5/a, 5/b, 5/c, where the running mean values of height reductions δ*g* referring to *1m* height are presented as the function of repetition number *n* for both the linear and the quadratic approximations.

Table 5/b. Variation of height reductions as the function of repetition number of measurements at h = 50, 200, 700, 1300 mm heights

	linear		quadratic	
n	δg	m	δg	m
1	–0.2514	±0.0050	–0.2565	±0.0038
2	–0.2504	±0.0049	–0.2543	±0.0044
3	–0.2511	±0.0056	–0.2539	±0.0044
4	–0.2508	±0.0055	–0.2539	±0.0044
5	–0.2506	±0.0053	–0.2536	±0.0043
6	–0.2504	±0.0056	–0.2536	±0.0044
7	–0.2504	±0.0053	–0.2532	±0.0042
8	–0.2500	±0.0057	–0.2528	±0.0048
9	–0.2506	±0.0056	–0.2535	±0.0048
10	–0.2508	±0.0055	–0.2535	±0.0046
11	–0.2511	±0.0056	–0.2542	±0.0048
12	–0.2511	±0.0055	–0.2542	±0.0046

Table 5/c. Variation of height reductions as the function of repetition number of measurements at h = 50, 300, 900, 1300 mm heights

	linear		quadratic	
n	δg	m	δg	m
1	–0.2549	±0.0162	–0.2627	±0.0084
2	–0.2532	±0.0124	–0.2577	±0.0074
3	–0.2546	±0.0106	–0.2560	±0.0069
4	–0.2540	±0.0102	–0.2553	±0.0067
5	–0.2536	±0.0093	–0.2545	±0.0063
6	–0.2542	±0.0094	–0.2540	±0.0066
7	–0.2544	±0.0093	–0.2540	±0.0065
8	–0.2546	±0.0088	–0.2544	±0.0062
9	–0.2545	±0.0084	–0.2553	±0.0058
10	–0.2542	±0.0085	–0.2546	±0.0059
11	–0.2541	±0.0085	–0.2547	±0.0060
12	–0.2539	±0.0083	–0.2547	±0.0059

Comparing the data referring to different height combinations in Table 5/a, 5/b, and 5/c we can see, that differences of height reduction values computed by linear approximation are bigger than computed by quadratic approximation for the same repetition number (8 μGal in the linear and 2 μGal in the quadratic case). Furthermore, it can be seen from data in Table 5/b and 5/c, that increasing the repetition number over four, both the height reductions δg and their standard deviations m do not vary significantly, so improvement of these values is not worth the extra work. It is probable that the reliability of measurements at higher levels may decrease a little bit because of the measuring tripod's vibration-sensitivity.

Finally, height reduction values computed by quadratic approximation are summarized in Table 6 referring to the measurements at three and four different heights in our height interval between 50 and 1631 mm.

Table 6. Height reductions in μGal by quadratic approximation referring to measurements at three and four different heights, and the max. height reduction differences between them

h [mm]	ABC	ABCD/1	ABCD/2	ABCD/3	max.diff.
50	13.8	**13.0**	13.0	**13.1**	0.8
100	27.5	26.0	25.9	26.4	1.6
200	54.7	51.9	51.6	**52.4**	3.1
206	**56.3**	52.9	**53.2**	53.4	3.4
300	81.6	**77.6**	77.4	78.2	4.2
400	108.2	103.2	102.9	103.8	5.3
560	150.0	143.3	**143.3**	143.9	6.7
600	160.4	154.3	153.4	154.6	7.0
700	186.1	179.7	178.5	**179.7**	7.6
800	211.5	205.0	203.5	204.6	8.0
900	236.5	**230.3**	**228.4**	229.4	8.1
911	**239.3**	232.3	231.1	232.0	8.2
1000	261.3	255.3	253.2	253.9	8.1
1100	285.8	280.4	277.8	278.3	8.0
1200	309.9	305.4	302.3	302.5	7.7
1300	333.8	**330.3**	326.7	**326.5**	7.3
1631	**410.6**	412.1	**406.6**	404.6	7.5

In Table 6 bold numbers indicate the heights where the measurements were made. Based on data in Table 6 it can be stated that height reductions computed by quadratic approximation from measurements at both three (ABC) or four (ABCD/1, /2, /3) different heights differ from each other. Moreover, these differences are the biggest in the crucial part of height-interval between 800 and 900 mm, where most absolute gravimeter's reference heights can be found.

5 Conclusions

Based on the above discussion and data several important conclusions can be drawn with respect to the determination of vertical gradients of gravity.

i.) Measuring only by one relative gravimeter either at two or at more different heights, it is impossible to reach the ±1μGal accuracy of height reduction because of the size of regular and random errors of measurements.

ii) Measuring a large number of series (increasing the repetition number over 4) by a single gravimeter does not give significant improvement of VG's value. Increasing the repetition number over four, both the height reductions and their standard deviations do not vary significantly, so improvement of these values does not worth the extra work.

iii) Several μGal differences are possible between linear or quadratic approximations, differences of height reductions may be approximately 13 μGal at the height of *0.8-1.0 m*. This is very important in the case of absolute gravity measurement's reduction, because most absolute gravimeter's reference heights are in the same interval. Based on our experiences the best height reduction values come from the measurements by three gravimeters, measuring 3-4 series at four different heights, and applying the quadratic approximation for processing the measurements.

iv) In case of reduction of measured gravity from the reference height of an instrument to the bench mark it would be expedient to apply a standard method to measure and determine the vertical gradient, because the computed values depend on the number of applied gravimeters, the repetition number of measurements, the heights of measurements, and the computation method (linear or quadratic approximation).

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