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Remeasurement of the Eötvös-experiment, status and first results

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This work was supported in part by the National Research, Development and Innovation Office – NKFIH 124286 and 124366. (Hungary).

Preparations of remeasurement of the Eötvös-Pekár-Fekete (EPF) experiment for the Weak Equivalence Principle were reported in [4]. Here we give a brief overview of the gravity field bias and try to estimate its possible effect on the EPF experiment. We report our first test results in the solar gravity field with Cu-Au and Al-Au pairs. The estimated errors were at level $2 \cdot 10^{-9}$, the same accuracy that was obtained by EPF. We also found that angular positions of the balances showed good correlation with bandpass-filtered air pressure variations hence Wiener filtering may significantly reduce this correlated noise. We did not detect any deviation from the equivalence principle considering the estimated measurement error.

International Conference on Precision Physics and Fundamental Physical Constants - FFK2019
9-14 June, 2019
Tihany, Hungary

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1.Introduction

Roland Eötvös and his colleagues Dezső Pekár and Jenő Fekete executed a series of measurements (EPF experiment) from 1906 till 1908 to validate the equivalence of gravitational and inertial mass [1]. Almost 80 years later, in 1986, Fischbach et al [2] reanalyzed their data and found a systematic violation that was only a couple of times larger, than the uncertainty of the measurement. Unfortunately, this violation has not been reproduced or explained by any later, more accurate, but partially different measurement methods. However, the an EPF violation effect is not completely excluded [10] and also there are some new experiments in preparation where the understanding of the working conditions of Eötvös-type torsion balances could improve the performance and sensitivity [9].

Via our new analysis of the EPF data, we have found a possible source of systematic error [3] that justifies repeating the experiment under better conditions and using current technology. The measurements are executed in the Jánossy Underground Physics Laboratory (JUPL) at Wigner Institute Budapest, at a depth of 30m, under suitable, controlled conditions [4, 5]. The Eötvös Year (the 100th anniversary of Roland Eötvös's death) ensures special attention to the new measurements. In our current study, we briefly review the history and report the preparation and current status of the measurements.

2.Principle of the EPF experiment and gravity gradient bias

The main idea of the EPF experiment [1] was to compare the horizontal component of gravitational force acting on a mass m as mG due to the Earth acting on different materials or samples with the horizontal component of centrifugal force mC (Fig.1). Centrifugal force is assumed composition independent, hence if gravitational force depends on material composition, the imbalance of horizontal forces can be detected with a torsion balance. If angle ε is the direction difference between gravity force mg (sum of gravitational and centrifugal forces) and gravitational force mG, $mG\sin\varepsilon$ is the horizontal component of this force. At geodetic latitude φ the horizontal component of centrifugal force is $mC\sin\varphi$. EPF introduced the Eötvös parameter η to characterize possible composition dependence of the gravitational force through formula $(1+\eta)mG$, assuming $\eta=0$ for a selected reference material. This parameter is the ratio of the horizontal component of the differential acceleration of the upper and lower masses of the balance and the horizontal component of the gravitational acceleration. EPF worked with 10 pairs of samples. The effect on the samples below the arm was compared to the fixed upper mass by the Eötvös parameter η. The results of EPF tests were finally described in terms of variation of the parameter between different pairs of samples. If there was no bias, nonzero parameter variation indicated equivalence principle violation.

Since the small force to be detected is in the North-South direction, the arm of the balance must be set to the East-West direction for maximum effect. (For direction reference we will consistently use the position of the lower mass of the balance.) In East-West direction of the arm (half length l) the assumed composition dependence of gravitational forces leads to torques of the same magnitude $\Delta\eta mGl\sin\varepsilon$ but with opposite sign. Differential rotation v_1 of the arm is proportional with differential torque on the balance which is rotated by 180° in the East-West direction. This differential torque is $2\Delta\eta mGl\sin\varepsilon$, and the constant of proportionality is reciprocal of the torsion constant τ of the fiber. If v_1 is measured, the difference of η can be calculated

$$\Delta\eta = -\frac{\tau v_1}{2mGl\sin\varepsilon}\,. \tag{1}$$

Spatial variation of the gravitational force must also be considered. A local North-East-Down reference frame is used: the x axis points to North, y to East and z to Down. In this frame only the x-component of the gravitational force, g_x exerts torque on the balance in East-West position. Variation of this force between the masses in East and West positions in linear approximation is $mg_x(z) = mg_{xz}z$, where g_{xz} is the vertical gradient of g_x. Differential rotation due to gravitational gradients is thus expressed as

$$v_2 = -\frac{2}{\tau} mlhg_{xz}\,, \tag{2}$$

where h denotes vertical distance between upper and lower masses.

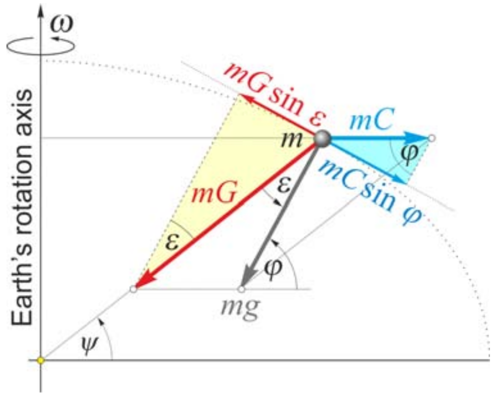

Fig.1. *Principle of the EPF experiment*

Formula 2 clarifies the two important requirements to be met to avoid gravity gradient bias in the EPF experiment. We see that v_2 should be kept strictly constant during the measurement. Otherwise change of the total differential rotation angle $v = v_1 + v_2$, according to Eq.(1) might be interpreted as a false violation of the equivalence principle.

First, torsion constant τ, mass m, half arm length l and vertical mass distance h should either be the same for the sample pair or should be measured and used for correction. Since constancy of τ cannot be assumed but its variation cannot be measured accurately, Eötvös and his coworkers used a smart idea to get rid of its change. They made use of the fact that there is no torque due to the composition dependent gravitational force on the balance in the North-South direction, but there is a gradient effect causing a differential rotation w very similar to Eq.(2)

$$w = -\frac{2}{\tau} mlhg_{yz}, \tag{3}$$

due to vertical gravity gradient g_{yz}. (We note that Eqs. (2) and (3) differ in sign since forces mg_x and mg_y with positive sign cause opposite torques.) The ratio v/w is free of the critical parameter τ but $\Delta\eta$ can still be calculated from the change of this ratio across different samples if gravity gradients were unchanged. This was their Method 2 [1].

The second requirement was that ambient gravity gradient g_{xz} (and in Method 2 also g_{yz}) must be unchanged during the experiment. To get rid of this requirement and still avoid bias EPF took simultaneous measurements with a pair of samples using a double balance. Hence any possible change of gradients had the same effect on the sample pair; by differencing v/w across the two balances at the same time these effects disappeared. After the first set of measurements they measured a second set by exchanging the samples between the two balances to cancel any false effect coming from the slightly different parameters and orientation of the individual balances of the double balance. This was their most advanced Method 3. It must be noted, however, that the output of their experiment, $\Delta\eta$ for measured sample pairs, contained results obtained with both methods [1].

Now we consider the origin of a gravity gradient bias that was not recognized by EPF. Equation (2) is valid both for point masses and for homogeneous circular cylinders if l and h refer to their centers of mass. The latter can easily be verified by integration.

However if the vertical variation of g_x is not strictly linear, the next possibility is to use the quadratic approximation $g_x(z) = g_{xz}z + g_{xzz}z^2$. The total gravitational force must be calculated by integration of $g_x(z)$ over the cylindrical samples used by EPF.

$$v_2 = -\frac{2}{\tau}\int_{z_1}^{z_2} m_z l\, \mathrm{g}_z(z)dz \tag{4}$$

where z_1, z_2 denote heights of upper and lower faces of the cylindrical sample and m_z is mass of infinitesimal cross section. An easy calculation for a sample with height $H = z_2 - z_1$ results in

$$v_2 = -\frac{2}{\tau} ml \left(h g_{xz} + \left(h^2 + \frac{H^2}{12} \right) g_{xzz} \right) \tag{5}$$

Equation (5) points to a new source of gravity gradient bias in the EPF experiment. It is due to sample height dependence. If sample height varies from H to H' and g_{xzz} is nonzero, there is a gravity gradient bias

$$\Delta\eta_{bias} = -\frac{g_{xzz}}{12 G \sin\varepsilon} \left(H^2 - H'^2 \right) \tag{6}$$

in the Eötvös parameter, which translates to a false violation of the equivalence principle. EPF used samples with very different heights in their experiment. For example the height of the Pt cylinder was 6 cm, that of Magnalium (Mg-Al alloy) was 11.9 cm and their Snakewood cylinder was 24 cm long. (We remark that Eq.(5) is valid only for thin cylinders. A better approximation contains a term proportional to $H^2/12 - R^2/4$ [3]. This expression depends on the radius R of the cylinder as well, but our conclusion on the origin of the bias remains the same.)

Now we estimate the magnitude of the bias. According to Eq. (6) it is proportional with g_{xzz}, the coefficient of the quadratic term. Close to surfaces of density jumps there are strong nonlinearities of g_x. Since the original field books, notes and possible sketches of the EPF measurement are unavailable, we can only guess what masses were close to the balances at the measurement site(s).

According to [2], the probable site of the EPF experiment was a small building with four windows facing South. We constructed a simple mass model of the building made of bricks. As it can be seen on Fig.2, calculations with this mass models showed that g_{xzz} may easily be as big as 0.2 nGal/cm² inside the building. This corresponds to a gradient bias in $\Delta\eta$ $1\cdot10^{-9}$ in case of the Magnalium-Pt sample pair. This bias is comparable with the results $\Delta\eta = \pm1\text{–}6\cdot10^{-9}$ reported by EPF [1]. We recently also measured $g_{xzz} = 0.07$ nGal/cm^2 in the Jánossy Underground Physics Laboratory (JUPL) with an improved Pekár G-2B torsion balance [4, 5].

Fortunately the above gravity gradient bias can be avoided in a remeasurement by carefully selecting sample shapes. Indeed, if the quantity $H^2/12 - R^2/4$ is kept constant for the samples, the gravity gradient bias is identically zero with no dependence on the local gravity field.

Although this hypothesis does not explain the correlation found by Fischbach et al.[6], we can make an interesting observation on Methods 2 and 3 used by EPF. Their published results were obtained initially with Method 2 and later with their refined Method 3. In Method 2 there was one balance only, sample pairs were measured subsequently. In Method 3 both balances of a double balance were used and sample pairs were measured simultaneously. Hence, temporal gravity field variation may have significantly biased Method 2, but not Method 3. If our hypothesis is true, we must see the sample geometry effect in Method 3 results but we may not see this effect in Method 2 results.

The bias is linearly dependent on the $\Delta q_{31}/q_{31}$ multipole moment ratio between the samples [3]. Fig.3. shows an approximate linear depencence of Method 3 results, in agreement with our hypothesis.

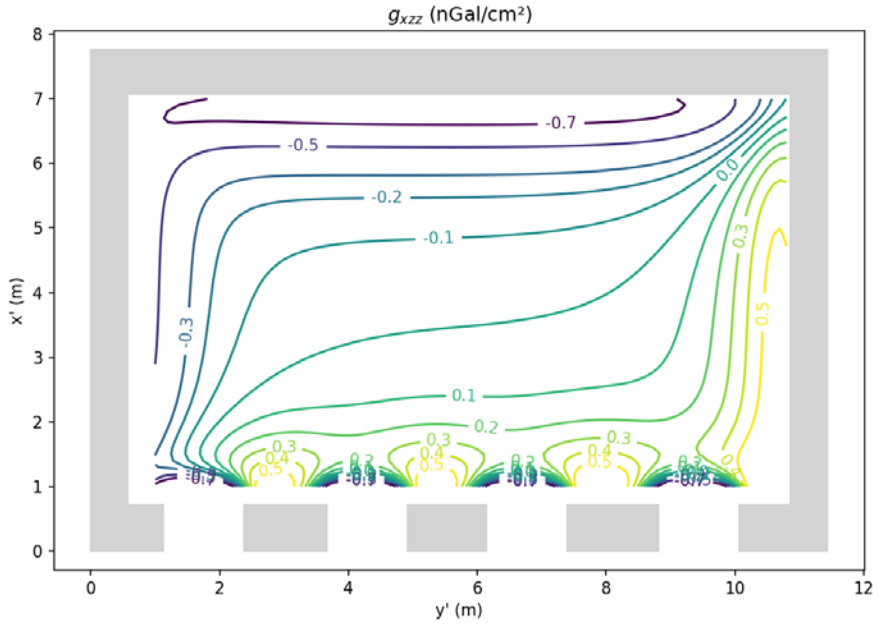

Fig.2. *Quadratic term in the variation of* $g_x(z)$ *from mass modeling at the probable site of EPF measurements*

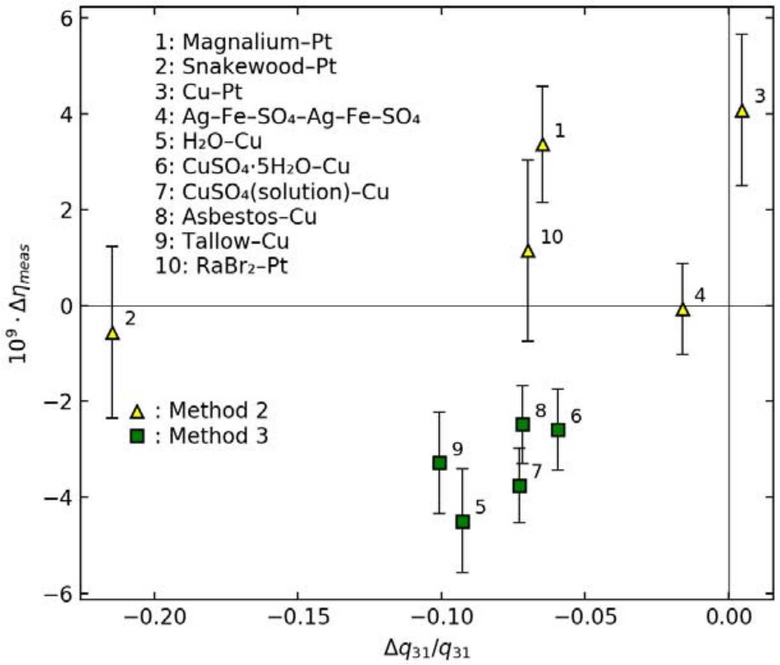

Fig.3. *Variation of the Eötvös parameter* $\Delta\eta$ *as a function of relative multipole change* $\Delta q_{31}/q_{31}$ *of the balance for sample pairs. Approximate linear dependence is seen for Method 3 results, which supports the hypothesis of gravity field originating bias in the experiment.*

3. First test results in the solar gravity field

Tests in the solar gravity field do not require rotation of the balance. For maximum effect, the beam should be positioned in the North-South direction of the local meridian. EPF tested a Magnalium-Pt pair in the gravity field of the Sun and reported for this pair $\eta = 2 \cdot 10^{-9}$ [1].

We first tested a Cu-Au pair in Sun's gravity field. The upper masses of both balances of the Pekár balance were Au, since those cannot be replaced. We measured during 6.5 days in 0° azimuth (North) with Cu-Au, and 13 days with significant gaps in 180° azimuth (South) with Cu-Au. Fig.4 shows the cumulative PSD of observed angular position residuals. Strong signal contribution can be seen in the 5 min - 3 h range, which is centered on the damped torsional eigenfrequency of the balance (30 min). Sun's gravity was next tested on Al-Au pair with 6 days in North azimuth with Al-Au pair. Quadratic drift was removed from the data that were downsampled to 1 min.

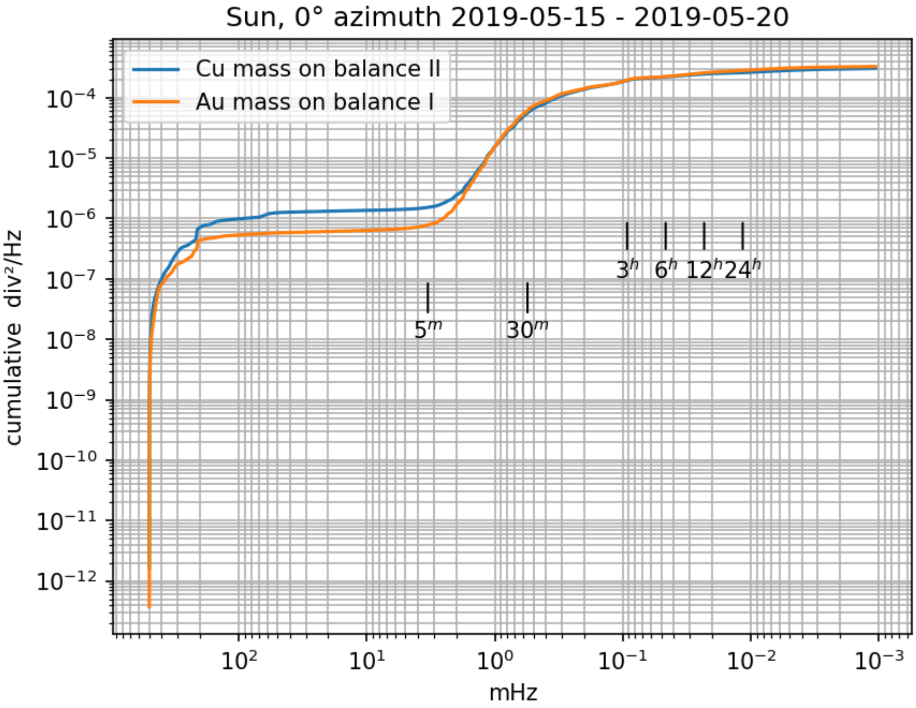

Fig.4. *Cumulative PSD of angular positions of an original double Pekár balance G-2B measured at the North azimuth of the balance's arm. The two graphs shows results with lower masses Cu and Au. The upper masses of both balances were Au.*

Estimation of the equivalence parameter in the solar gravity followed the method used by Roll, Krotkov, Dicke [7]. First, least-squares fitting was performed to readings n as function of true local solar time t ($t = 0$ at noon) according to the formula

$$n(t) = S\,sin(t) + C\,cos(t) + K \tag{7}$$

Equivalence parameter η was calculated from the amplitude S of the sine term in Eq. (7). Table 1. shows the calculated equivalence parameters. From non-null results on Au-Au pairs of identical material, the estimated errors are at level $2{\cdot}10^{-9}$. We note that this is already the same accuracy that was obtained by EPF.

Tab.1. *Calculated equivalence parameters for solar gravity tests for specific measurement periods. Materials of lower masses are indicated. Upper masses were Au in each case.*

measurement period	material	η
05.15-05.20.	Au–Cu	$0.60{\cdot}10^{-9}$
05.15-05.20.	Au–Au	$1.15{\cdot}10^{-9}$
05.21-06.04.	Au–Cu	$-1.85{\cdot}10^{-9}$
05.21-06.04.	Au–Au	$-1.01{\cdot}10^{-9}$
06.04-06.11.	Au–Al	$1.50{\cdot}10^{-9}$
06.04-06.11.	Au–Au	$1.86{\cdot}10^{-9}$

Initial quadratic drifts of both balances were about 0.01 div/h (scale division per hour) after wire loading, but became almost linear after 1 day and decreased to at least 0.005 div/h (2 µrad/h). Drift corrected residuals of the two balances showed puzzling (anti)correlation. Temperature fluctuations were small, the variation was ±0.02°C under undisturbed conditions of the measurement site (heat dissipation caused by human activity can produce much larger temperature fluctuations). Finally it turned out that these correlated variations are due to pressure changes. Bandpass filtered (2 min - 60 min) air pressure variations generally show good (anti)correlation with angular position of the balances (Fig.5), in agreement with findings at Tula University with wideband gravitational gradiometers [8].

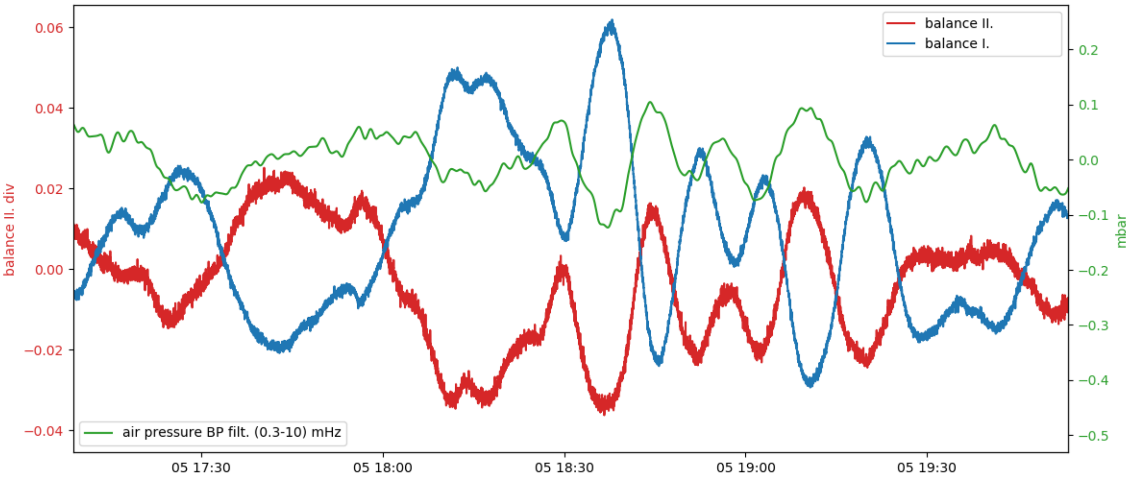

Fig.5. *Angular positions of balances and 0.3-10 mHz bandpass-filtered air pressure variations. Air pressure variations were measured in situ with a Bosch BME280 sensor and show good (anti)correlation with measured angular positions.*

The effect strongly depends on the azimuth of the balance's arm. One possibility to reduce noise is Wiener filtering of the pressure corelated fluctuations, and this improvement in data processing is planned for the future.

4.Summary

In the EPF experiment Roland Eötvös and his coworkers determined the error of the equivalence of inertial and gravitational masses so that it was larger than the errors of the individual measurements. In 1986 E. Fischbach and his cooworkers explored a systematic deviation in the measurement. The origin of this deviation is still mysterious [6]. According to our hypothesis gravity gradient effects could be a reason, becasue the original samples of Eötvös were cylinders with uniform diameter and higher order gravity gradient originated force depends on the shape of the samples. This possible systematic error in the EPF measurements can be checked by repeating the experiment with better conditions and modernized technology. Therefore, in 2017 it has been decided that the Eötvös experiments were repeated with an original but improved Eötvös balance. A research group was formed with researchers from MTA Wigner Research Centre from Physics, from the Departments of Geodesy and Surveying and of Control Engineering and Information Technology of the Budapest University of Technology and Economics and also from the Society for the Unity of Science and Technology among others.

Our modernized small original Pekár type Eötvös balance started the weak equivalence principle tests in summer 2019 in the Jánossy Underground Physics Laboratory (JUPL), -30 meters underground. Here at the conference we have reported the results of the first four weeks. Also in this case the Wiener filtering of the correlated pressure fluctuations is necessary to reduce noise, and finally we did not detect any deviation from the equivalence principle considering the estimated measurement error.

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