Different adjustment methods for the Hungarian part of the unified European Gravity Network

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Abstract The first version of the Unified European Gravity Net (UEGN) was set up in 1993, based on the participation of 11 countries. The aim was basically the generation of the unified European geodetic basement for global geologic and geodynamic purposes. The unified scale of the network is ensured by numerous absolute gravimetric stations. Since 1995 further countries (inter alia Hungary) have been joined to the unified network. In this paper different adjustment methods for the Hungarian net part is presented by the authors. Robust procedures are described besides the least squares method, as both a free and a constrained network. Our methods and software are ready to process all the European gravity data.
Keywords: Hungarian Gravity Network, Europen Gravity Network, adjustment of gravity network, robust adjustment.

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1 Introduction

The International Union of Geodesy and Geophysics (IUGG) has long been planning to set up a unified scale and datum gravimetric network which could be applicable in the whole continent of Europe. Its conditions have been established by now, because several countries have got portable absolute gravimeters (AXIS, JILAG, etc.), providing unified scale in accordance with the current accuracy specifications. At the same time the need for increasing the accuracy of global geodetic reference systems, the solving of several geodynamic and geotectonic problems, have brought about the realisation of this objective as a daily routine.
As regards the number of absolute stations and point density (the number of 1st and 2nd order bases as well as their regional distribution), and also their accuracy, the Gravimetric base networks of the individual European countries are rather heterogeneous. It seems both necessary and expedient to establish a unified network whose principles were recommended at the joint conference of the Geodesy and Geophysics Working Group (GGWG) of NATO and the Mapping Services of East European Armies held in Budapest in collaboration with civil experts in 1994. The essence of it defines the creation of a network consisting of absolute points at a general distance of 100–150 km within which the 1st and 2nd order bases are expedient to be measured with modern relative gravimeters.
The US National Imagery and Mapping Agency (NIMA, formerly DMA) began to increase the accuracy of WGS-84 reference ellipsoid in 1991 and substantially helped Hungary establish both a Hungarian national Military GPS Network (KGPSH) and an absolute gravimetric base network Ádám et al., (1994).

2 The Hungarian Base Network (MGH-2000) – the UEGN part of the network

When establishing Hungary's new gravimetric base network, we considered the following aspects as important:

- Over the course of the planning of zero order base net (absolute stations) we attributed great importance to placing them evenly all over the country as well as setting them quite close to GPS geodynamic points, which had been established some time before, so that an economical system of integrated networks could be set up. We call these points integrated network points which means that the same point is a member of GPS, gravity, levelling, etc. base network.
- When new points are established, or the destroyed ones are replaced, one has to bear in mind the changes of ownership which is closely related to the protection of points.

MGH–2000 is part of the joint networks of the three countries Czech Republic, Hungary and Slovakia (UGN), so the methods of modern network planning could partly applied Csapó, Sárhidai (1985), because the joint form is basically determined by the previous and applicable parts of UGN. In any case, we have experienced that the planning of networks for optimal network measurements can give rise to the necessity of establishing connections between far away points. However, this might not be carried out (transporting instruments by planes) on account of Hungary's present financial situation.

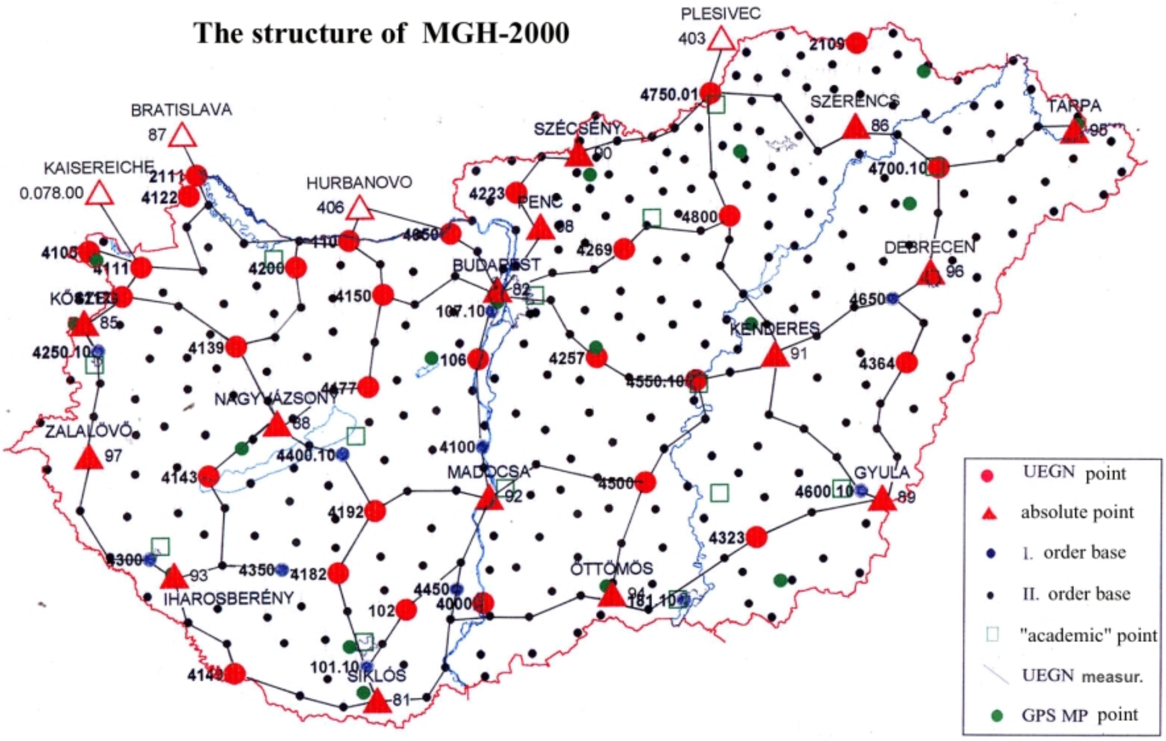

Fig. 1. The Hungarian Base Network MGH-2000 and its UEGN part

2.1 The zero order network

The use of such network is meant to ensure the scale of the national (entire) base network as well as checking the stability of gravity by repeated observations. The zero order network consists of 15 absolute stations (6400km^2/point) their location is given in Fig. 1, including foreign absolute stations near the borders as well. These points were placed at the ground level of significant buildings whose survival and accessibility seem to be ensured for a long time (manorhouses, mansions, etc.). Monumentation was implemented by floor level 120 by 120 by 100 cm concrete blocks. A brass bolt was fixed to the middle of the upper level of the block indicating the height above sea-level according to the Baltic system. The points were tied to two or three points of the national levelling network, allowing ± 5 mm accuracy.

The geographical coordinates of the stations was determined on the basis of 1:10000 topographic maps with ± 1 second confidence limit. Gravity values relating to the reference heights of absolute gravimeters was determined with LCR gravimeters allowing 1,5–3 μGal confidence limit. The station established in Budapest is of extraordinary importance, because measurements have repeatedly been carried out on it with absolute gravimeters of different type in every two or three years since 1980. At most stations repetitive measurements were carried out in the past three or four years as well. Each station has got at least one "excenter point" which is monumented with a concrete block of 80 by 80 by 100 cm outside the building. The relative confidence limit of its *g* value is not worse than 5 μGal.

2.2 First order network

The 19 points included in *Fig. 1* are by and large the same as the bases of MGH–80 placed at airports Csapó, Sárhidai (1990). The distance varies between 50 to 70 km and the density of points is 4400 km2/point. The determinations of geographical coordinates of the points was similar to the methods described in section 2.1. Altitude determination was done with 1–10 mm confidence limit.

2.3 Second order network

As mentioned already, these points were established by ELGI in the 1970s. The distance between the individual points is 10 to 15 km in hilly areas, whereas it ranges between 15 to 25 km in the plains. The average density of points is 250km^2/point. We have replaced a couple of dozens of points which were destructed during the last twenty years and have integrated them into MGH-80. The new network contains 386 second order points.

On *Fig. 1*. the points of MGH-2000 are presented together with the points which are members of UEGN-2000.

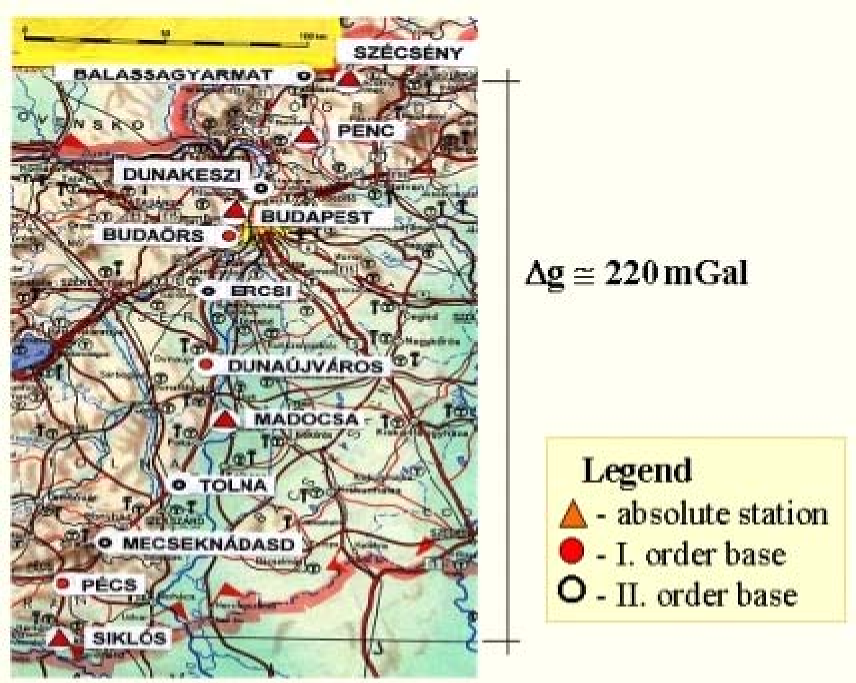

Fig. 2. The map of the national gravimetric calibration line

3 National gravimetric calibration line

Its present form had been developed since 1969, its present status was accepted in 1985. Formerly it served as the scale of the Potsdam Gravity System (the first absolute station was established in Budapest in 1980). There are five absolute stations within the 210 mGal range of the line (the highest Δg value is 250 mGal between the base points of the country). The other points of the calibration line are 1st and 2nd order points with an average distance of 30 km from each other (*Fig. 2*). Δg values between the points were previously determined with Askania Gs–12, GAG–2, Sharpe, Worden, then LCR gravimeter groups. The vertical gradients of the points were determined with a group of 3-4 LCR instruments with an accuracy of 4–7 μGal Ccapó (1987). The relative accuracy of each point is 8–12 μGal. The calibration line is part of UGN, and the section of Siklós-Budapest is the southern part of the *Carpathian Polygon*. The Carpathian Polygon was established by ELGI in collaboration with Czech, Polish, and Slovakian partners in 1973 to monitor the non–tidal variations of the gravity field in the Carpathians (the line starts from Siklós absolute point then goes via Budapest, Zilina, Zakopane to Krakow). It was reobserved in 1978–79, 1988–89 and 1999-2000.

4 Adjustment of measurements by the Least Squares method

The observed data can be adjusted by the least square method as a constrained network. The fixed points of the network may be the latest *g* values of the absolute gravity stations, Csapó and Sárhidai (1990). The mean value of observed gravity difference (Δg) observed in *A–B–A–B–A* system by one gravimeter (which means the average of the four observed difference) can be taken as one individual measurement.

Taking into account the large number of unknowns and the problems of numerical stability of adjustment computations, the matrix orthogonalization method can be used for practical adjustments, Völgyesi (1979, 1980, 2001). The base principle of matrix orthogonalization method can be demonstrated by the hyper-matrix transformation:

$$\begin{bmatrix} \underset{(n,r)}{\widetilde{\mathbf{A}}} & \underset{(n,1)}{\widetilde{\mathbf{l}}} \\ \\ \underset{(r,r)}{\mathbf{E}} & \underset{(r,1)}{\mathbf{0}} \end{bmatrix} \rightarrow \begin{bmatrix} \underset{(n,r)}{\widetilde{\mathbf{W}}} & \underset{(n,1)}{\widetilde{\mathbf{v}}} \\ \\ \underset{(r,r)}{\mathbf{G}^{-1}} & \underset{(r,1)}{\mathbf{x}} \end{bmatrix}$$

where

$$\underset{(n,r)}{\widetilde{\mathbf{A}}} = \underset{(n,n)}{\mathbf{P}^{1/2}} \underset{(n,r)}{\mathbf{A}} \quad ,$$

$$\underset{(n,1)}{\widetilde{\mathbf{l}}} = \underset{(n,n)}{\mathbf{P}^{1/2}} \underset{(n,1)}{\mathbf{l}} \quad ,$$

$\mathbf{A}$ is the coefficient matrix of the observation equations, $\mathbf{l}$ is the vector of absolute terms, $\mathbf{P}$ is the weight matrix, $\mathbf{E}$ is a unit matrix, $\mathbf{0}$ is a zero vector; $\mathbf{W}$ is a matrix having orthogonal columns, and $\mathbf{G}^{-1}$ is an upper triangular matrix, n is number of equation, r is number of unknowns, Völgyesi (1980).

This matrix transformation directly yields the unknowns x_i in place of vector $\mathbf{x}$, variances and covariances of unknowns x_i are comprised in weight coefficient matrix

$$\mathbf{Q}_{(\mathbf{x})} = \mathbf{G}^{-1}(\mathbf{G}^{-1})^*$$

where $(\mathbf{G}^{-1})^*$ is transposed of $\mathbf{G}^{-1}$.

After executing transformation the corrections v_i can be computed from the $\widetilde{\mathbf{v}}$ vector of transformed hyper-matrix, using the equation

$$\underset{(n,1)}{\mathbf{v}} = \underset{(n,n)}{\mathbf{P}^{1/2}} \underset{(n,1)}{\widetilde{\mathbf{v}}} \quad .$$

In case of practical computation each columns of the hyper-matrix should be stored one by one on hard disk. Since the actual transformation is being performed in the RAM of a computer, the maximum number of equations and unknowns is limited by the RAM size (free place for at least two columns must be provided in the RAM at the same time). Matrix-orthogonalization method gives a good possibility to solve large equation systems in general RAM size beside high numerical stability, Völgyesi (2001).

5 Robust adjustment methods

5.1 Adjustment of measurements by minimizing the L_1 norm of the correction vector

Since our observations are contaminated by noise, usually more measurements are carried out than the number of parameters to be defined. In these cases one has to handle over-determined or mixed-determined problems. The correction vector is not a null vector, therefore the system of equations to be solved are inconsistent. In this case several solutions exist, which are generally based on the minimization of some norms of the correction vector.

Various types of optimization processes were developed in order to give the most reliable estimation for the parameters. Gauss introduced the least squares (LSQ) method based on the L_2 norm of residuals. Laplace applied the minimization of the sum of absolute deviations (L_1 norm). The choice among the different norms depends on what kind of weight should be given to the data which are relatively far from the trend (outliers).

The LSQ procedure based on the minimization of L_2 norm leads to optimal parameter estimation in case the measurement noise follows Gaussian statistics (normal distribution). In most cases this condition is not fulfilled. In practical applications very often wider tail-distributions have to be supposed (e.g. because of the outliers). The most frequently supposed example is the simple exponential (Laplace) distribution Menke (1984), where the minimization of the objective function using the L_1 norm of the correction vector $\mathbf{v}$ (the sum of absolute deviation of measured and observed data)

$$\sum_{i=1}^{n} |v_i| \rightarrow \min \tag{1}$$

leads to optimal estimation.

Besides its inevitable advantages and simplicity, the greatest disadvantage of the LSQ method is being extremely sensitive to the outliers, as it redistributes the locally occurring outlier errors to all the deviations, causing reliability problems in case of contaminated, noisy set of observations.

In such cases robust procedures are applied, which are less sensitive to the divergence from the normal distribution, therefore they reduce the biasing effect of the relatively large errors. One of the most frequently applied procedures is one based on the minimization of the L_1 norm of the correction vector.

The principle of the adjustment based on the L_1 norm is briefly the following. One can describe the intermediary equations between the y_i observations (measured data) and the x_j parameters to be defined by the adjustment and furthermore the a_{ij} coefficients by the expression Závoti (1999)

$$y_i = \sum a_{ij} x_j + v_i \quad . \tag{2}$$

The unknown vector of parameters $\mathbf{x}$ has to be estimated in the way that the objective function

$$\Phi(\mathbf{x}) = \sum_{i=1}^{n} |\nu_i| = \sum_{i=1}^{n} \left| y_i - \sum_{j=1}^{m} a_{ij} x_j \right| \qquad (3)$$

is to be minimized.

One can get the solution of the extremum problem by deriving the (3) objective function accordant to the values of $\mathbf{x}$, where the

$$\frac{\partial \Phi(\mathbf{x})}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\sum_{i=1}^{n} \left| y_i - \sum_{k=1}^{m} a_{ik} x_k \right| \right) = 0 \,, \quad j = 1 \ldots m \qquad (4)$$

system of equations can be set up. Performing the operations one can get the equations

$$\begin{aligned} \frac{\partial \Phi}{\partial x_j} &= -\sum_{i=1}^{n} \mathrm{sgn}(\nu_i) \sum_{k=1}^{m} a_{ik} \delta_{kj} = \\ &-\sum_{i=1}^{n} \frac{1}{|\nu_i|} \left(y_i - \sum_{k=1}^{m} a_{ik} x_k \right) a_{ij} = 0 \end{aligned} \,. \qquad (5)$$

By introducing the elements of diagonal weight matrix $w_{ii} = 1/|\nu_i|$ eq.(5) can be taken to the form

$$\sum_{i=1}^{n} \sum_{k=1}^{m} a_{ik} w_{ii} a_{ij} x_k = \sum_{i=1}^{n} a_{ij} w_{ii} y_i \,, \quad (j=1..m), \qquad (6)$$

or else the normal set of equations of the problem is

$$\mathbf{A}^{\mathbf{T}} \mathbf{W} \mathbf{A} \mathbf{x} = \mathbf{A}^{\mathbf{T}} \mathbf{W} \mathbf{y} \qquad (7)$$

in matrix form (e.g. Scales et al. 1988), where

$$\mathbf{W} = diag\{\{1/|\nu_i|\}\} \,, \quad (i=1 \ldots n).$$

The resulting set of equations – as usually the normal system of equations of robust procedures – is non-linear. Most frequently the numerical problems occurring at solving the non-linear set of equations are evaded by the help of more complicate iterative procedures. This kind of procedure is e.g. the IRLS (Iteratively Reweighted Least Squares) algorithm, where the solution of the non-linear equations is lead back iteratively to that of linear set of equations. In that case the appropriately weighted form of the linear system of equations is solved in each iteration step. After the determination of the weight matrix one can apply a simple, iteratively reweighted linear LSQ algorithm.

The substance of the algorithm is that in the first iteration step the solution corresponding to the L_2 norm is derived, then in each following step the

$$\mathbf{W}^{(0)} = \mathrm{diag}\{\{1/\nu_i^{(0)}\}\},$$

where

$$\nu_i^{(0)} = \left| y_i - \sum_{k=1}^{m} a_{ik} x_k \right|$$

diagonal weight matrix is calculated from the data of the previous iteration step. Thus one can get the

$$\mathbf{A}^{\mathbf{T}} \mathbf{W}^{(0)} \mathbf{A} \mathbf{x} = \mathbf{A}^{\mathbf{T}} \mathbf{W}^{(0)} \mathbf{y}$$

linear set of equations. The characterizing step of jth iteration is

$$\mathbf{A}^{\mathbf{T}} \mathbf{W}^{(j-1)} \mathbf{A} \mathbf{x} = \mathbf{A}^{\mathbf{T}} \mathbf{W}^{(j-1)} \mathbf{y} \,. \qquad (8)$$

Consequently, in each iteration step the weighted form of (8) linear set of equations has to be solved by applying the weight matrix $\mathbf{W} = \mathbf{W}^{(j-1)}$. The procedure is being performed until an appropriately chosen stop criterion is fulfilled.

Kis (1998) introduced a generalized objective function in which λ^2-fold of the L_q norm of the relative parameter vector was added to the L_p norm of the relative data deviation vector

$$\Phi = \|\mathbf{v}\|_p + \lambda^2 \|\mathbf{x}\|_q = \sum_{i=1}^{n} \left| y_i - \sum_{k=1}^{m} a_{ik} x_k \right|^p + \lambda^2 \sum_{k=1}^{m} |x_k|^q \qquad (9)$$

This objective function (with p and q parameters) is suitable for solving over- and mixed-determined problems. A generalized adjustment procedure applying the IRLS method is defined throughout the minimization of this objective function with the typical jth step

$$\left(\mathbf{A}^{\mathbf{T}} \mathbf{W}^{*(j-1)} \mathbf{A} + \frac{q\lambda^2}{p} \mathbf{R}^{*(j-1)} \right) \mathbf{x} = \mathbf{A}^{\mathbf{T}} \mathbf{W}^{*(j-1)} \mathbf{y} \qquad (10)$$

in which the proper choice of p and q parameters leads to some of the most frequently used adjustment procedures (LSQ [$q = p = 2$, $\lambda = 0$], Marquardt-Levenberg [$q = p = 2$, $\lambda \neq 0$], LAD-IRLS [$p = 1$, $q = 0$]), and some new IRLS procedures can also be defined. In eq.(10)

$$\mathbf{R}^* = diag\left\{\left\{|x_k|^{q-2}\right\}\right\}, \quad (k=1 \ldots m),$$

and

$$\mathbf{W}^* = diag\left\{\left\{|\nu_i|^{p-2}\right\}\right\}, \quad (i=1 \ldots n)$$

are the weight matrices. In the special case of $q = p = 2$, the $\mathbf{R}^*$ and $\mathbf{W}^*$ are unit matrices, eq.(10) is a linear set of equation, therefore the IRLS method is not required for the solution.

By the modification of the simplex method Barradole and Roberts (1973) elaborated an effective method to solve the linear programming problem

$$\mathbf{Ax} = \mathbf{y} + \mathbf{v}, \quad \sum |v_i| \rightarrow \min,$$

$$x_j = x_j^+ - x_j^-, \; v_i = v_i^+ - v_i^-,$$

$$\mathbf{Ax}^+ - \mathbf{Ax}^- - \mathbf{v}^+ + \mathbf{v}^- = \mathbf{y},$$

$$\sum v_i^+ + v_i^- \rightarrow \min .$$

Závoti (1999) expounds a two-phase simplex algorithm. In the first phase a base solution of the linear programming problem is searched. In the second phase the solution of the over-determined set of equations is given using the simplex table resulted from the first phase as initial data.

Procedures minimizing the L_1 norm are less sensitive to noisy data sets, consequently in case of the adjustment of data sets containing outliers much favourable results can be achieved then minimizing the L_2 norm.

5.2 The Danish method

The method was developed by Krarup et al. (1980) in order to decrease the effect of relatively large errors in the adjustment. At the first step of the procedure the adjustment is carried out by using the LSQ method, and all observed data has equal weight ($p = 1$). Afterwards the weight matrix is determined iteratively:

$$p_{i,j} = \frac{1}{1 + a_k v_{j-1}^2} \quad ,$$

where j is the actual iteration step. The a_k coefficient is correct if p=0.25 for the erroneous measurement. The threshold of errors can be taken as the function of the errors of unit weight, then:

$$a_k = 3 / v_k^2$$

where

$$v_k = 3\mu_0 \quad \text{if} \quad v_{\max} > 3\mu_0$$

$$v_k = 2\mu_0 \quad \text{if} \quad 2\mu_0 < v_{\max} < 3\mu_0$$

$$v_k = \mu_0 \quad \text{if} \quad \mu_0 < v_{\max} < 2\mu_0$$

Csapó and Völgyesi (2001). The erroneous measurements will get less weight by each subsequent iteration step. The iteration should be reiterated until the error of unit weight is decreasing in a considerable way.

In the adjustment of MGH–2000 two steps of iteration proved to be sufficient. The data set consisted of all the measurements of MGH–2000, absolute points near to the border in the neighbouring countries and connecting ties across the borders, altogether 5544 individual observed gravity differences and their corrections, 450 unknowns (point values and scale factors of gravimeters) 20 absolute points, 436 gravimeter points including 8 Austrian and 42 Slovakian ones.

Now our methods and software are ready to process UEGN data.

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