INVESTIGATION OF HUNGARIAN TORSION BALANCE MEASUREMENTS BY PREDICTION

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ABSTRACT

Torsion balance measurements in Hungary were checked by least-squares collocation. The methodology was the so-called "leave-one-out" prediction of horizontal gravity gradients. The method was successfully tested on a selected subset of 700 torsion balance measurements and only three possible outliers has been detected. These results are promising in view of a planned new Hungarian geoid determination.

Keywords: torsion balance measurements, prediction, gravity field

INTRODUCTION

Altogether about 60000 torsion balance stations were measured by the institutions MAORT, ELGI and OKGT in Hungary in the XXth century (Szabó, 1999). Unfortunately due to different reasons a certain part of the measurements are missing today, however, the remaining part can still be saved for applications. In the last 10 years measurements at 24450 were processed and saved into a computer database at ELGI (Eötvös Loránd Geophysical Institute). Although measurements in the past were mainly collected for prospecting of oil and gas, at a significant part of the measurement points besides horizontal gravity gradients W_{zx}, W_{zy} also curvature values W_Δ and W_{xy} have been determined, and what is more, largely topographic effects were also computed. The distribution of torsion balance measurements stored in the computer database is shown in Fig. 1.

What is the accuracy of these data? A possible answer to this question can be deduced from repeated measurements. Since this is not possible to us, however, we would like to use these data in gravity field modeling and for a new Hungarian geoid solution, another approach can be followed. The idea is that we attempt to "reproduce" any measurement from its neighbors. Any prediction process in principle could be used for the purpose, on the other hand it is customary for data validation, at least in gravity field modeling, to use least squares collocation proposed by Moritz (1980). In this paper this method is reviewed for validation of Hungarian torsion balance measurements. The principles of the method is demonstrated test computations with this method are presented.

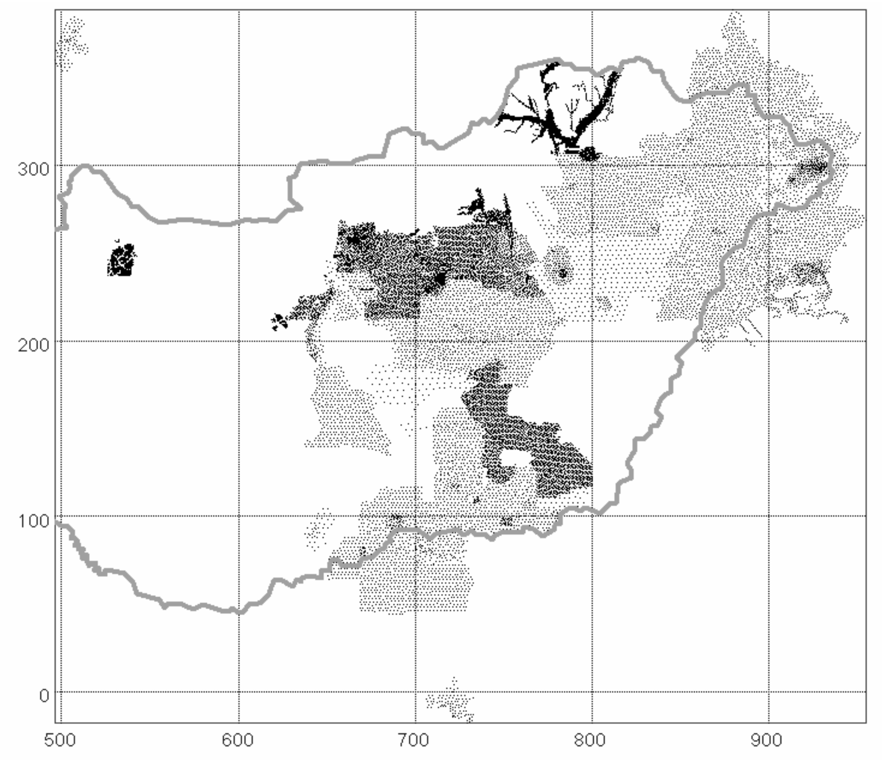

Fig. 1. Status of the computer database of stored torsion balance measurement points. The coordinates are in km unit in the Hungarian national EOV grid.

DATA VALIDATION BY LEAST SQUARES PREDICTION

To validate torsion balance data the method of LOO (Leave One Out) prediction was applied. The idea of the method is to make prediction at each torsion balance station from the *neighborhood* of this point (of course without using the measurements of the point itself). After this the difference between prediction and actual measurements of the point is compared to the error of the prediction. This process is repeated for each measurement point and we will then see whether there are any statistically significant outliers in view of the prediction error.

The equations of least squares prediction are reviewed for example by Moritz (1980). The main relation is well-known:

$$s = C_{sl}(C_{ss} + C_{nn})^{-1}\ell , \qquad (1)$$

where ℓ is the vector of measurements, s is the result of prediction (signal) for a known/unknown station, C_{ss} is the signal, C_{nn} is the noise covariance matrix and C_{sl} is the cross-covariance matrix of measurements and predictions.

The covariance matrices in Eq. (1) can be deduced from theoretical auto- and crosscovariances using the distances between points for isotropic quantities, but we will need also the azimuths between any pair of points for anisotropic quantities like for example the torsion balance measurements. Hence in our case the necessary auto- and crosscovariances $C_{Wxz,Wxz}(t,a)$, $C_{Wyz,Wyz}(t,a)$ and $C_{Wxz,Wyz}(t,a)$ can be written as functions of distance *t* and azimuth *α*.

The theoretical covariance model proposed by Reilly (1979) was used. For us now the following auto- and crosscovariances are required, since we will use only the horizontal gravity gradients in the subsequent computations.

$$C(T_{xz},T_{xz}) = \frac{1}{2}\Phi(5,0) + \frac{1}{2}\Phi(5,2)\cos 2\alpha$$
$$C(T_{yz},T_{yz}) = \frac{1}{2}\Phi(5,0) - \frac{1}{2}\Phi(5,2)\cos 2\alpha$$
$$C(T_{xz},T_{yz}) = -\frac{1}{2}\Phi(5,2)\sin 2\alpha$$

The computation of functions $\Phi(p,q)$ can be found in Reilly (1979), and it involves the numerical evaluation of the Kummer hypergeometric functions as well as the definition of covariance function parameters C_0 and *d*.

The above parameters of the covariance model can be determined from isotropic empirical covariance function of gravity gradients (Tóth et al. 2005). This empirical covariance function with its standard deviations and the approximate covariance model can be seen in Fig. 2. We note, however, that this empirical function was determined from *all* available gradient data and since there are more than 1 billion possible combinations of the 40000 measurement points – to speed up the computation – it was necessary to pre-sort the data according to the coordinates. It is obvious that the covariance model fits the empirical data well only for distances between 0.8-4 km, but this is satisfactory for the present data validation purpose. On the other hand the robustness of collocation is well-known, i.e. the result is not too much dependent on the details of the chosen covariance function.

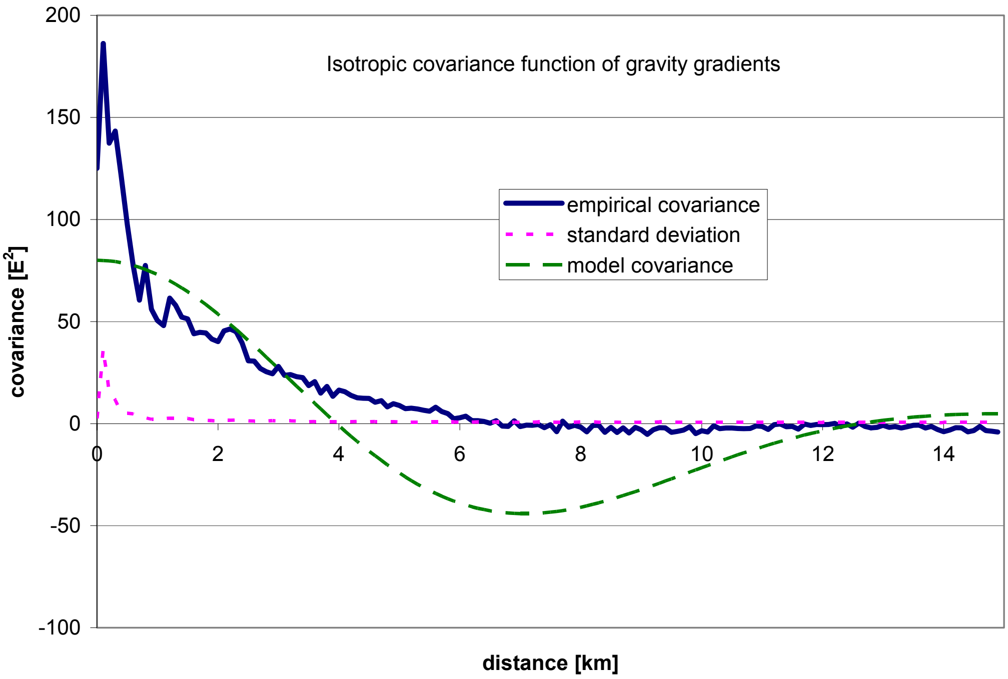

Fig. 2. Isotropic covariance function of gravity gradients

The first tests were performed at 100 selected stations near Tiszakécske. In Fig. 3 some of the results are shown: the original and predicted gradient vectors as well as their differences, and three-sigma confidence ellipses. To the standard deviation of gravity gra-

dient measurements the value ± 3 E (Eötvös; 1E = 10^{-9} s^{-2}) was assigned and the maximum point selection neighborhood (from where points were selected for prediction) was set to 1.5 km. It can be seen in Fig. 4 that at almost 18 percent of points the deviations exceeded the three-sigma threshold.

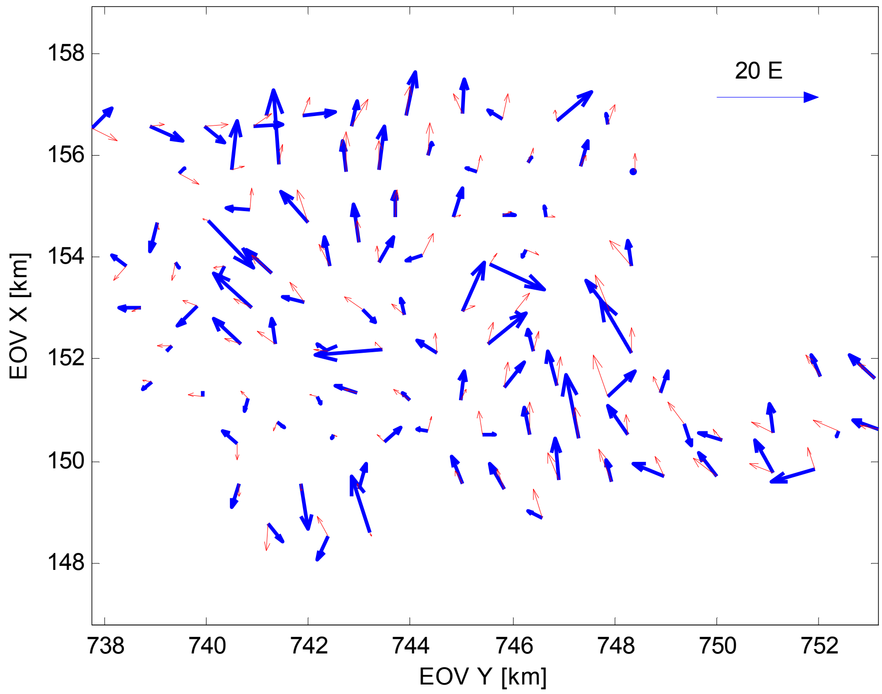

Fig. 3. Original and predicted gravity gradients. Predictions are shown with thick arrows.

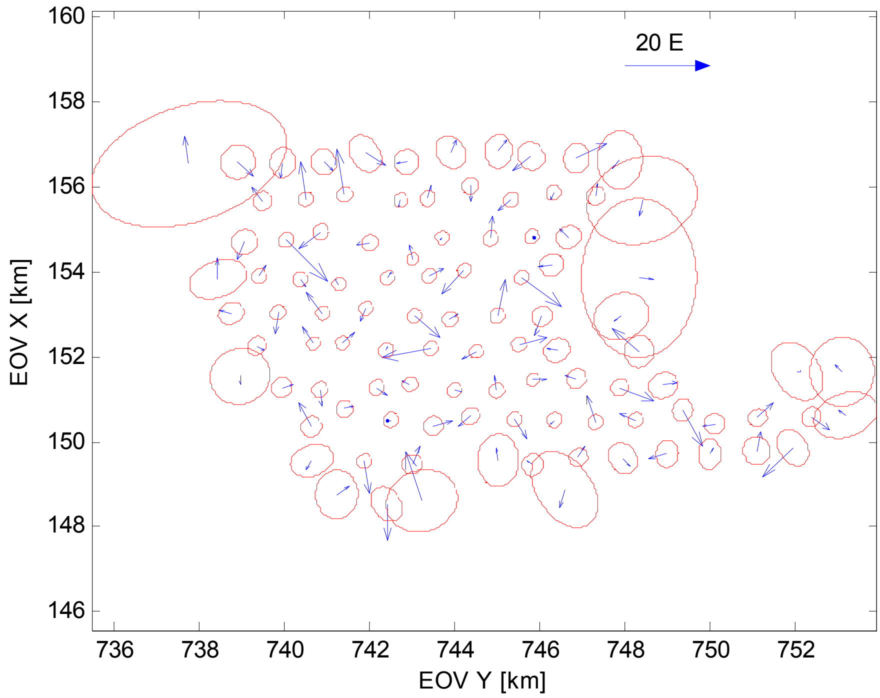

Fig. 4. Prediction error vectors and confidence ellipses for 100 stations near Tiszakécske.

The results above indicated that further tests are necessary. Therefore we repeated our test for 700 measurement sites to discover the effect of changing the standard deviation of horizontal gravity gradients on data validation. As it can be seen in Fig. 5, by increasing standard deviation of the measurements to ± 5 E only four points were found such the deviations exceeded the prescribed threshold of three times the prediction error; and even at these three sites only the W_{zx} component. Moreover, the resulting mean error ± 4,5 E is in good agreement with the apriori value ± 5 E. It can be concluded that in any case one should check the measurements at these four sites with possible outliers.

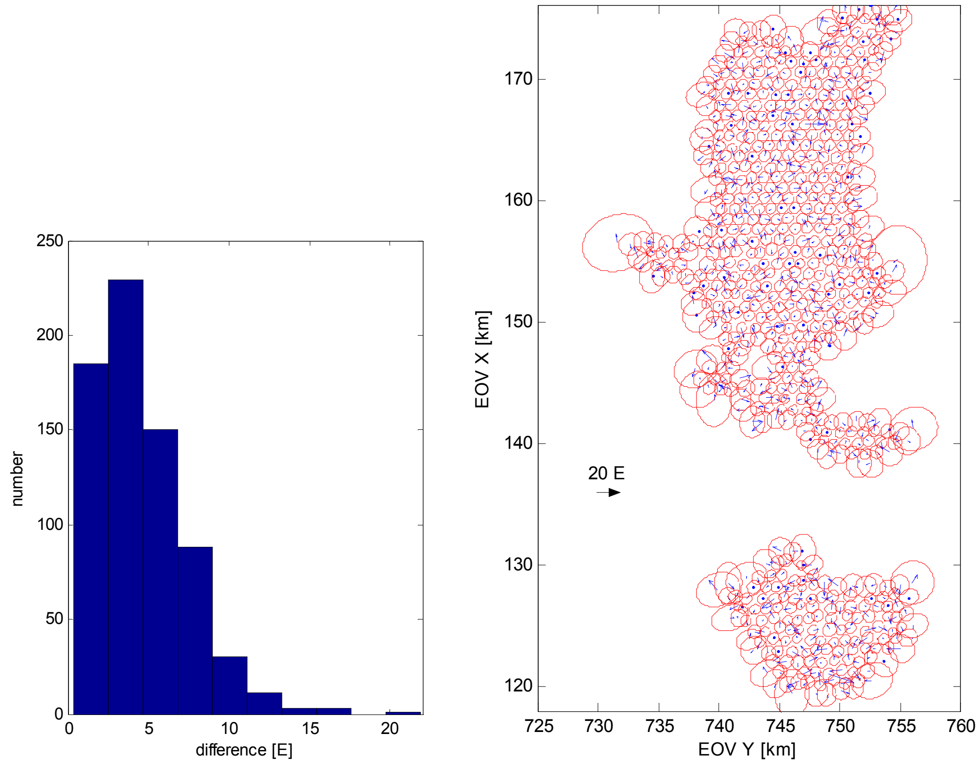

Fig. 5. On the left subfigure histogram of the deviations between measurements and predictions is shown (the mean deviation is 4.5 E). On the right subfigure one can find the vectors of prediction errors and three-sigma confidence ellipses for selected 700 torsion balance measurement sites.

The above method thus seems to work and it can be useful for further checking the database of torsion balance measurements.

GEOID DETERMINATION BY LEAST-SQUARES COLLOCATION IN HUNGARY

We are planning further tests by extending the validation procedure of this study to curvature values. Furthermore it is desirable to use also other kind of auto- and cross-covariance models, which fit better to the structure of the gravity field in Hungary.
The present study was initiated with a new Hugarian geoid soulution in mind, i.e. to use the extremely valuable torsion balance measurements for such a solution. It is planned also that we are able use point gravity data in the framework of the existing cooperation

between our university and ELGI. Furthermore the lithospheric model of the Pannonian Basin (Papp et al. 1996) as well as surface rock densities could be used to make gravity field data statistically more homogeneous.

Our next tests will aim at the computation of gravity anomalies Δg by collocation from gravity gradients using all existing measurements. In view of the large number of existing gravity gradients (the present number is about 50 thousand) the size of the covariance matrix of measurements would be about 15 GB, stored in double precision. By considering points within a distance of 6 km only, however, a *sparse matrix* can be obtained with 0.4% fill-in, and the size is thus reduced to 278 MB. Due to the fact that the horizontal gravity gradient is a 2-element vector, a 2x2 block matrix of measurement covariances will be yielded, and one such block with nonzero elements can be seen in Fig. 6. This covariance matrix is of manageable size and one can employ in-core and out-core sparse solvers (e.g. the 32-bit sparse solver library TAUCS, see Rotkin and Toledo, 2004).

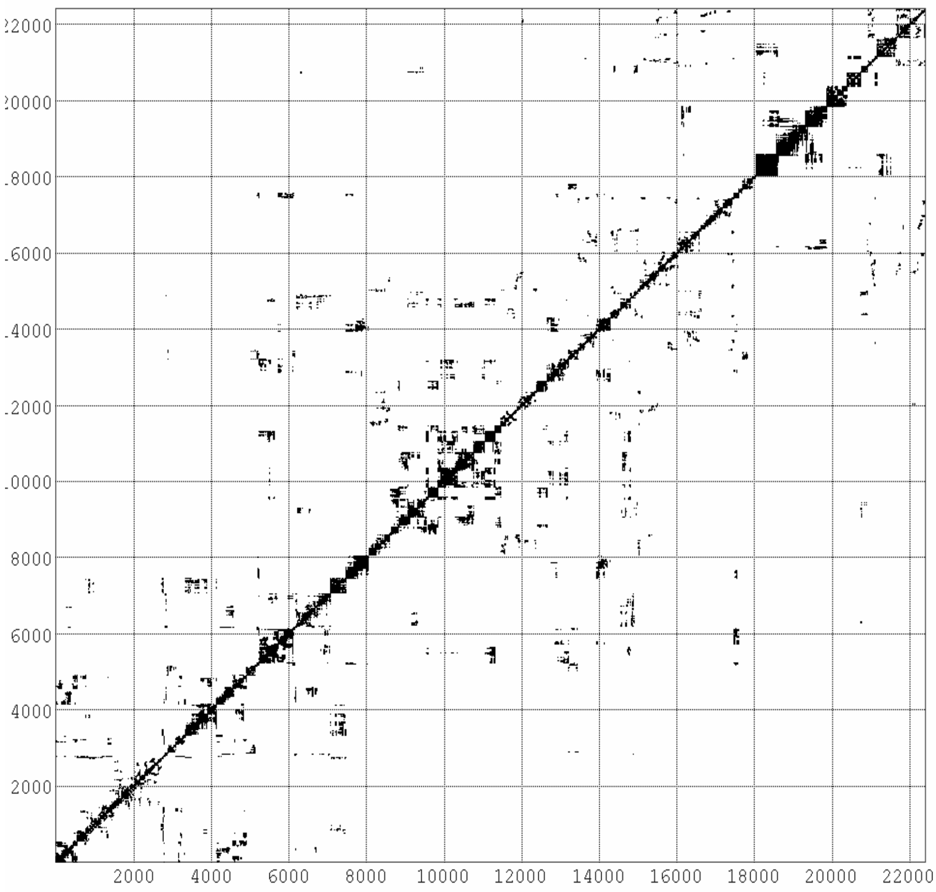

Fig. 6. Sparse covariance matrix block of gravity gradients

SUMMARY

Our test has shown that least-squares prediction may be suitable to validate the full database of Hungarian gravity gradients (where the sites are closer to each other than 4 km). After the validation process is finished, we would like to use these data in the near future for a new detailed and accurate local geoid determination.

ACKNOWLEDGEMENTS

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