NEW MEASUREMENTS FOR THE DETERMINATION OF LOCAL
VERTICAL GRADIENTS

G. Csapó (1), L. Völgyesi (2)

(1) Eötvös Loránd Geophysical Institute of Hungary, H-1145 Budapest,
Kolumbusz utca 17/23. E-mail: csapo@elgi.hu
(2) Budapest University of Technology and Economics, H-1521 Budapest,
Müegyetem rkp. 3. E-mail: volgyesi@eik.bme.hu

ABSTRACT

Measured vertical gravity gradients usually quite differ from the normal value of 0.3086
mGal/m (1mGal = 10^{-5} m/s^2). Generally the changing of vertical gradient is rather big,
up to a few 10 cm height above the ground and can be taken into consideration as a sec-
ond order function. Over 40 m height the changing is linear and over 500 m height the
changing is very small, the measured value of vertical gradient is about 0.3073 mGal/m.

At present the datum level (mGal level) of modern gravity networks are mostly de-
termined by absolute gravity values measured by absolute gravimeters, which values
are referred to the reference heights of absolute gravimeters. So the datum level (mGal
level) of the network of absolute points differs from the datum level of the network of
relative measurements. The vertical gradients can be used for the conversion of meas-
ured gravity from the reference height of an instrument to a bench mark. So the vertical
gradients are playing a key role for joining the two types of network. The height correc-
tion is necessary to determine by an accuracy of several µGal, not to decrease the reli-
ability of the transformed value of gravity. So the vertical gradients should be deter-
mined as high accuracy as it possible and using the normal value of vertical gradient
(0.3086 mGal/m) is not sufficient for this purpose.

Keywords: vertical gravity gradients, height correction, transformation between ab-
solute and relative gravity networks

1. DETERMINATION OF VERTICAL GRADIENTS

Importance of exact local value of vertical gradients (VG) was announced in earlier
publications (CSAPÓ 1997; CSAPÓ, PAPP 2000) both from the point of view of precise
gravity measurements and from the checking of values computed by model computa-
tions. Both mass inhomogenities of surrounding environment and the type of approxi-
mation method of function dg/dh have significant effects for the measured vertical gra-
dient values. It is pointed out, that measuring the vertical gradient only at two points
along the perpendicular line of a given point does not provide enough accuracy on the
one hand, and the measured vertical gradient values significantly differ from the nor-
mal value of 0.3086 mGal/m on the other hand (CSAPÓ, VÖLGYESI 2002).
Generally the changing of vertical gradient is rather big, up to a few 10 cm height above the ground and can be taken into consideration as a second order function, but over 40 m height the changing of $VG$ can be taken into account as a linear function (Röder, Wenzel 1986). According to the aerial measurements in the height interval of 500-1700 m the changing of vertical gradient is very small (it is only 1-2%), the measured value is about 0.3073 mGal/m (Hammer, Gumert 1984).

Based on our earlier test measurements and experiences a method was processed for field measurements of vertical gradients. These measurements give a suitable accuracy for any practical purposes.

Two LCR-G gravimeters and a special tripod were applied for these measurements (Csapó, Völgyesi 2002). Heights of measurements have been setting as the sensing mass of instruments were at special heights above the benchmark (A = 50 mm, B = 700 mm and C = 1300 mm) (Becker et al 2002). Nine series were measured by each gravimeter with a sequence of A-B-C-B-A-B-C-B-A. All LCR gravimeters were calibrated at a calibration line. Periodic errors of gravimeters by 1 mGal at the processing of measurements were taken into account, and gravimeters were equipped by electronic levels. The readings were made by spur wheel and digital voltmeter connected to the output of Capacitance Beam Position Indicator (CPI). Interpolation method was used for three readings of spur wheel and digital voltmeter. Duration of measuring sequence for each gravimeter were about 60 minutes, measurements were made by two instruments, one after the other. The observations were repeated on three different days at each point therefore six independent measured sequences could be processed for every point. In this way the vertical gradients can be computed from the corrected relative values of gravity measured by LCR gravimeters at different heights above a benchmark. The applied corrections can be found in the references.

Measurements were made in the years of 2002-2003 at different sites of Budapest. Coordinates of points, height reductions supposing linear $VG(1)$ and quadratic $VG(2)$ vertical gradient can be found in Table 1. If the variation of gravity is a linear function of height is supposed to be linear, the height reduction $VG(1)$ can be computed by

$$VG(1) = \partial g / \partial h \cdot \Delta h .$$

If the variation of gravity is a quadratic function of height the height reduction $VG(2)$ can be computed by

$$VG(2) = \partial g / \partial h \cdot \Delta h + \partial^2 g / \partial h^2 \cdot \Delta h^2 .$$

In Table 1 the $VG(1)$ and $VG(2)$ are the height reductions referring to the height of 1m above the benchmark.

In spite of the few measurements can be stated that there are usually no significant differences between the linear and quadratic height reductions at some points, but there can be significant differences at other ones.

Especially striking poor reliability can be seen at point VT (Gellért-hegy) which point was located just above an urban water reservoir. The poor reliability can be explained by the significant daily water-level fluctuation in the water reservoir - the daily changing of water mass may be a few 10000 m$^3$ (Csapó, Szabó, Völgyesi 2003). Therefore this continuously changing water mass causes the bigger dispersion of vertical gra-
Gradient values at VT point measured at different days and different parts of the days, than at the other points.

Table 1. Data of measured points and the linear and quadratic height reductions ($\delta g_h$) in $\mu$Gal

<table>
<thead>
<tr>
<th>points</th>
<th>name of ponits</th>
<th>$\phi$</th>
<th>$\lambda$</th>
<th>H[m]</th>
<th>$\delta g_h$(1)</th>
<th>$\delta g_h$(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2143</td>
<td>Hármashatárhegy</td>
<td>47-33-23</td>
<td>19-00-10</td>
<td>463</td>
<td>-386.0 ± 10</td>
<td>-393.5 ± 5</td>
</tr>
<tr>
<td>2142</td>
<td>Táborhegy</td>
<td>47-33-01</td>
<td>19-00-31</td>
<td>412</td>
<td>-350.0 ± 14</td>
<td>-354.7 ± 7</td>
</tr>
<tr>
<td>2141</td>
<td>Remetehegy</td>
<td>47-32-39</td>
<td>19-00-39</td>
<td>354</td>
<td>-350.6 ± 6</td>
<td>-351.3 ± 5</td>
</tr>
<tr>
<td>2140</td>
<td>Szépvölgyi út</td>
<td>47-32-27</td>
<td>19-00-29</td>
<td>283</td>
<td>-284.9 ± 6</td>
<td>-288.2 ± 6</td>
</tr>
<tr>
<td>2139</td>
<td>Mátyáshegy</td>
<td>47-32-00</td>
<td>19-00-57</td>
<td>201</td>
<td>-258.7 ± 8</td>
<td>-259.0 ± 8</td>
</tr>
<tr>
<td>821</td>
<td>Rózsadomb</td>
<td>47-31-02</td>
<td>19-01-53</td>
<td>153</td>
<td>-304.2 ± 8</td>
<td>-306.0 ± 4</td>
</tr>
<tr>
<td>17</td>
<td>BME kert</td>
<td>47-28-50</td>
<td>19-03-35</td>
<td>105</td>
<td>-300.1 ± 6</td>
<td>-305.3 ± 4</td>
</tr>
<tr>
<td>VT</td>
<td>Gellérthegy</td>
<td>47-29-24</td>
<td>19-02-32</td>
<td>160</td>
<td>-326.9 ± 21</td>
<td>-332.8 ± 12</td>
</tr>
<tr>
<td>MET</td>
<td>Pestlőrinc</td>
<td>47-25-48</td>
<td>19-11-01</td>
<td>137</td>
<td>-303.3 ± 9</td>
<td>-303.1 ± 6</td>
</tr>
<tr>
<td>OMH</td>
<td>Németvölgyi út</td>
<td>47-30-00</td>
<td>19-00-01</td>
<td>142</td>
<td>-293.5 ± 6</td>
<td>-293.3 ± 5</td>
</tr>
</tbody>
</table>

It’s worth mentioning that previous vertical gradient measurements were made previously on two different heights at point 821 (Mátyáshegy) have given 262.5 ± 4 $\mu$Gal for the value of $VG$ (1).

An example can be seen in Table 2 for the results of different vertical gradient measurements on three different heights at point 2139 (Rózsadomb).

Table 2. Results of vertical gradient measurements of two LCR gravimeters at point 2139. Height reductions was computed for the height difference $\Delta h=1m$. All the values are in $\mu$Gal.

<table>
<thead>
<tr>
<th>measurements</th>
<th>LCR-1919</th>
<th>LCR-963</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta g_h$(1)</td>
<td>$\delta g_h$(2)</td>
</tr>
<tr>
<td>1.</td>
<td>-309.4</td>
<td>-312.7</td>
</tr>
<tr>
<td>2.</td>
<td>-301.7</td>
<td>-307.6</td>
</tr>
<tr>
<td>3.</td>
<td>-303.2</td>
<td>-307.4</td>
</tr>
<tr>
<td>mean (1.-3.)</td>
<td>-304.8 ± 6</td>
<td>-309.2 ± 2</td>
</tr>
<tr>
<td>average (1919 &amp; 963)</td>
<td>-304.2 ± 8</td>
<td>-306.0 ± 4</td>
</tr>
</tbody>
</table>

It can be easily seen, that there is no significant difference between the height reductions determined by linear or quadratic approximation, but the reliability of $VG(2)$ is better than of $VG(1)$ for both gravimeters and for the all measurements. Moreover it can be seen from Table 2 that the difference of height reductions determined from the measurements of different gravimeters can be as high as 7$\mu$Gal, independently of the approximation method. So this justifies that a single gravimeter does not give significant reliability of local vertical gradient’s values. The reason of the various discrepancies of
computed $VG$ values from different measurement’s series is the changing effect of the varying wind, air pressure, temperature, microseismic effects, etc. These enumerated effects may cause various drift of gravimeters therefore the measurements made by certain gravimeters on the same days may be more or less different. Similar problem may come from the transporting vibration of gravimeters, or the discrepancies of measurements may come from the reason of not taking into account the periodic errors of gravimeters by $1 \text{ mGal}$.

Results of two different adjustment versions of vertical calibration basis of Budapest can be seen in Table 3 measured in 2003. Fixed values of adjustment were the gravity of absolute station of Budapest (82) and the station of Budaörs airport (107.10). Vertical gradient values of these points had been determined earlier, before the adjustment of Hungary's New Gravity Base Network (MGH-2000). The height reduction of gravimeters have been taken into consideration using the normal value of vertical gradient ($0.3086 \text{ mGal/m}$) in the case of adjustment version “$A$”, but the measured value from observation on three different heights (supposing the vertical gradient $dg/dH$ as a linear function of height) in the case of adjustment version “$B$”.

<table>
<thead>
<tr>
<th>point</th>
<th>$A$ mGal</th>
<th>$VG$ mGal/m</th>
<th>$B$ mGal</th>
<th>$VG$ mGal/m</th>
<th>$\delta V (B-A)$ mGal</th>
</tr>
</thead>
<tbody>
<tr>
<td>2139</td>
<td>835.3946</td>
<td>–0.3086</td>
<td>835.3962</td>
<td>–0.3042</td>
<td>0.0016</td>
</tr>
<tr>
<td>2140</td>
<td>809.9906</td>
<td></td>
<td>809.9921</td>
<td>–0.2849</td>
<td>0.0015</td>
</tr>
<tr>
<td>2141</td>
<td>796.0111</td>
<td></td>
<td>796.0152</td>
<td>–0.3506</td>
<td>0.0041</td>
</tr>
<tr>
<td>2142</td>
<td>783.8461</td>
<td></td>
<td>783.8501</td>
<td>–0.3500</td>
<td>0.0040</td>
</tr>
<tr>
<td>2143</td>
<td>771.1283</td>
<td></td>
<td>771.1347</td>
<td>–0.3860</td>
<td>0.0064</td>
</tr>
<tr>
<td>82</td>
<td></td>
<td></td>
<td>–0.2519</td>
<td></td>
<td></td>
</tr>
<tr>
<td>107.10</td>
<td></td>
<td></td>
<td>–0.3084</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen from Table 3, that demonstrable differences of adjusted values of points - measured by high precision gravimeters and computed by the two types of height reduction values - are possible. There was only maximum $1 \mu\text{Gal}$ difference between the height reduction values computed by linear or quadratic approximation because of the small heights (50-120mm) of instruments.

2. ESTABLISHING THE CONNECTION BETWEEN ABSOLUTE AND RELATIVE GRAVITY NETWORKS BY VERTICAL GRADIENTS

The datum level ($\text{mGal}$-level) of modern gravity base networks is defined by gravity values determined by absolute measurements. These absolute points are the zero order network points in most of European countries, and form the framework of UEGN-2000 network too. Gravity values measured by absolute and relative gravimeters are referring to the reference height of the instrument above the bench mark. Each gravimeter has special reference height therefore the different networks of absolute and relative points have different datum level.

There are two possible methods of conversion between the two types of gravity networks.
In the case of first method the vertical gradient values referring to the absolute points must to be determined with some kind of procedure. The measured vertical gradient values depend partly on the method of determination partly on the approximation method of \( VG \) function (linear or quadratic) as the examples show in literature and in this paper. Differences of height reductions depending on the applied approximation method may often be larger than the accuracy of measurements by absolute gravimeters. At the same time application of \( VG \)'s normal value for reducing the measured gravity to the bench mark is not suitable, because according to the effect of mass inhomogenities next to the bench mark quite different \( VG \) value have to be applied. In this case the technical parameters of \( VG \)'s determination (linear or quadratic approximation method, instrument heights) have to be uniformly applied for all gravity points.

The other method of conversion between the two types of gravity networks is presented by the help of Fig. 1.

![Fig. 1. Definition of different heights for the conversion of g](image)

First of all a height should be chosen on the perpendicular line of point \( P \) which is the average reference height of absolute gravimeters. The reference height of absolute gravimeters can be found between about 80 and 110 cm depending on the construction of instruments, therefore \( H=100 \text{ cm} \) can be chosen for the average height (the distance between the point \( P \) and \( B \) is 100 cm). In practice the reference height of an absolute gravimeter is at the height of point \( C \). At the same time the reference height of a relative gravimeter is at the height of \( H_{mg} = 5 - 10 \text{ cm} \) therefore the sensing mass of the instrument is at the height of point \( A \) (there is no possibility to set up the gravimeters so that the level of sensing mass would be at the level of point \( P \)). Determining the vertical gradient by measurements, the sensing mass of the relative gravimeter is at this height of point \( A \) as well, at the lowest position of the instrument. Consequently the \( g \) referring to point \( C \) measured by absolute gravimeter can be convert to point \( P \) so that the gravity difference \( \Delta g \) should be determined between the point \( A \) and \( B \), then \( \delta g_1, \delta g_2 \) height correction should be added to it:
$g_p = g_C + \Delta g + \delta g_1 \pm \delta g_2$,

where $\delta g_1$ is the height correction between the level of the sensing mass of a relative gravimeter and the bench mark (between point $A$ and $P$), and $\delta g_2$ is the height correction between the real and the average reference height of absolute gravimeters (between the point $B$ and $C$). As the vertical gradient value has been determined by some kind of method, then height corrections of $\delta g_1$ and $\delta g_2$ can be computed as well. There is no significant difference larger than the demand of accuracy between the height reduction $\delta g_1$ computed by linear or quadratic approximation because the distance $PA$ is only 5-15 cm. The situation is the same with height reduction $\delta g_2$ because the height difference between the real and the average reference height of absolute gravimeters (between the point $B$ and $C$) is only 10-20 cm. So the maximum difference is 1 $\mu$Gal between the height reduction $\delta g_2$ computed by linear or quadratic approximation for the distance $BC$. Based on the decision of the specialists working on the adjustment of Unified European Gravity Net (UEGN-2000) we have applied this latter type of conversion.

ACKNOWLEDGEMENTS

Our investigations were supported by the Hungarian National Research Fund (OTKA), contract No. T 031875, T-037929 and the Physical Geodesy and Geodynamics Research Group of the Hungarian Academy of Science.

REFERENCES


Dr. Lajos VÖLGYESI, Department of Geodesy and Surveying, Budapest University of Technology and Economics, H-1521 Budapest, Hungary, Műegyetem rkp. 3. Web: http://sci.fgt.bme.hu/volgyesi E-mail: volgyesi@eik.bme.hu