

MODELING TIME VARIATION OF GRAVITY GRADIENTS DUE TO WATER LEVEL FLUCTUATIONS

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ABSTRACT

The water level fluctuations of the Danube River in Budapest during the great flood in 2002 were recorded. The 3D model of the river bed allowed us to build an accurate polyhedral model of the time variation of mass changes of the flood. This mass density variation model made it possible to compute and compare time variations of various gravitational field functionals. Gravity and full gravitational gradient tensor changes were computed on a regular grid for the model area and these changes were compared with accuracies attainable for gravity measurements. This way we were able to evaluate which kind of gravitational field parameter is more suitable for detecting gravitational field variations due to near-site mass changes. Gravity changes were also compared with actual gravitational field measurements made during the flood with two LCR gravimeters. We studied also the possibility of predicting gravity changes from certain combinations of measured gravity gradients where no mass variation model is available. This latter technique may be relevant in the future to support absolute gravity measurements by detecting the effect of local gravitational field changes through repeated gravity gradient measurements.

Keywords: time variation of gravity gradients, polyhedron density model, full gravitational tensor

1. THE GRAVITATIONAL EFFECT OF WATER-LEVEL FLUCTUATIONS OF THE DANUBE RIVER

In the summer of 2002 there was a great flood of Danube River and gravity measurements were carried out to study the effect of water level fluctuations of the water mass. One station was located on the right-hand embankment at Szabadság-bridge in Budapest, the other one about 500 m to the west, at the foot of Gellért-hill where the water of Danube had already no significant gravitational effect (see Fig. 1). Unfortunately the

gravity stations are influenced not only by the level of the Danube, but by ground-water fluctuations as well.

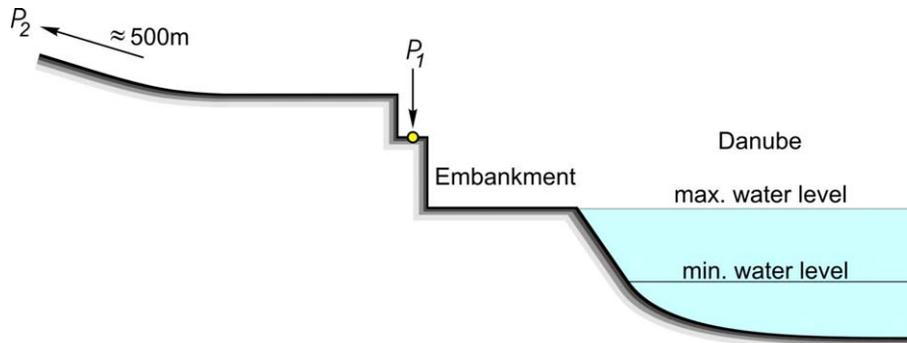


Fig. 1. Sketch for the measurements of Danube flood

Measurements were carried out by two LCR gravimeters on four subsequent days because of the fast recession of the flood. Daily observations were carried out in a P1–P2–P1–P2–P1–P2 sequence. The maximum subsidence of the level of the Danube was 4.16 m and the respective gravity change 41 μGal (CSAPÓ, SZABÓ, VÖLGYESI 2003).

To verify the measurements, model calculations were carried out applying the software Mod3D written by I. Cerovský (CEROVSKÝ ET AL, 2004). Mod3D is an interactive 3D geophysical gravity and magnetic modeling software, it has been developed to create 3D geophysical models in a user friendly interactive environment. Mod3D computes anomalous gravitational field components $\mathbf{g}(g_x, g_y, g_z)$ and the full gravitational tensor elements $W_{xx}, W_{yy}, W_{zz}, W_{xy}, W_{zx}, W_{zy}$. Anomalous gravitational fields are computed using formulae for polyhedral bodies. The application of polyhedron volume element can provide a more realistic geometrical description of boundary surface of a geological body (e.g. topographic surface, without height jumps) than the description made by rectangular prism (parallelepiped) models. Using the polyhedron density model the generated second derivatives of the gravitational potential are more smooth and realistic function than the ones provided by prism model (BENEDEK, 2002). If the calculation level is near to the surface of the gravitational source, the accuracy of related quantities of gravitation can be increased by a detailed description of this surface in the vicinity of calculation.

We have calculated the gravitational effect of the water mass of Danube flood using a three-dimensional model. The 3D model of the Danube flood can be seen in Fig. 2. The heights of topography and the position of water mass were given a 10×10 m grid spacing. Dimension of the model is 640×640 m. Anomalous gravitational field components and the gravitational tensor elements were computed along the central cross sectional part (*M section*) of this model.

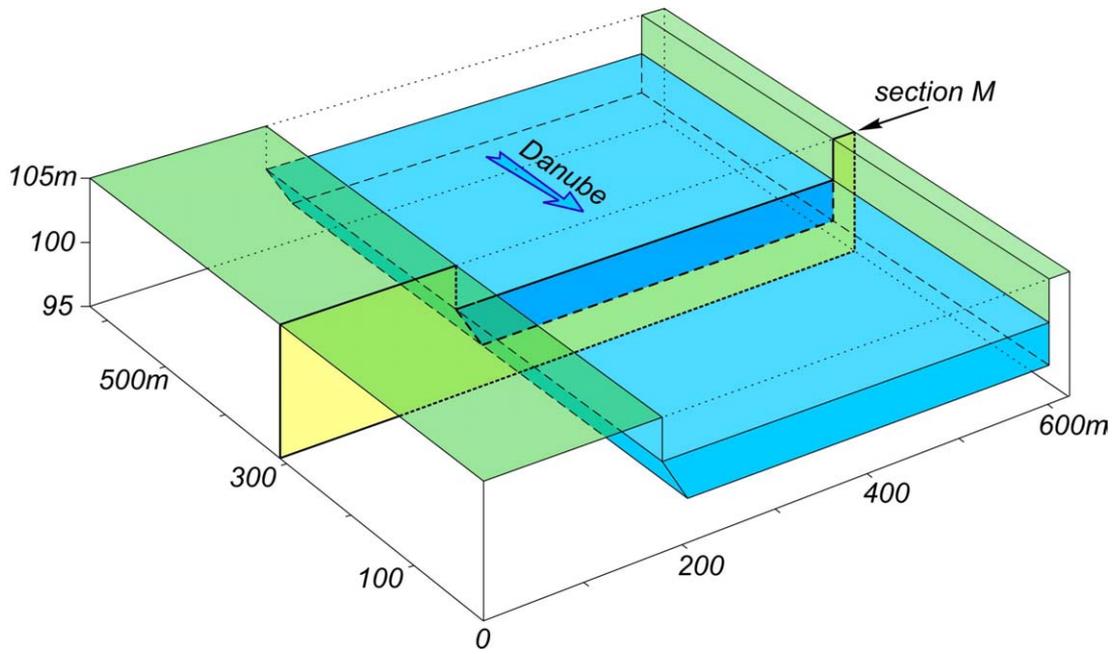


Fig. 2. The 3D model of water mass

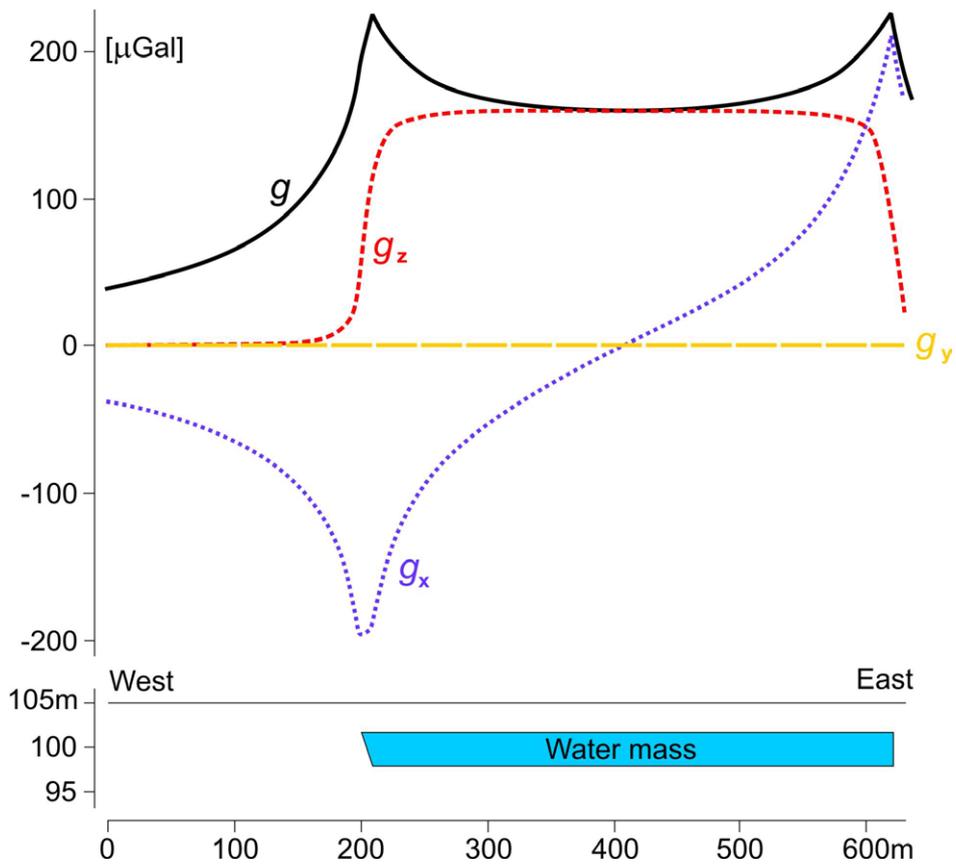


Fig. 3. The gravitational field components \mathbf{g} (g_x, g_y, g_z)

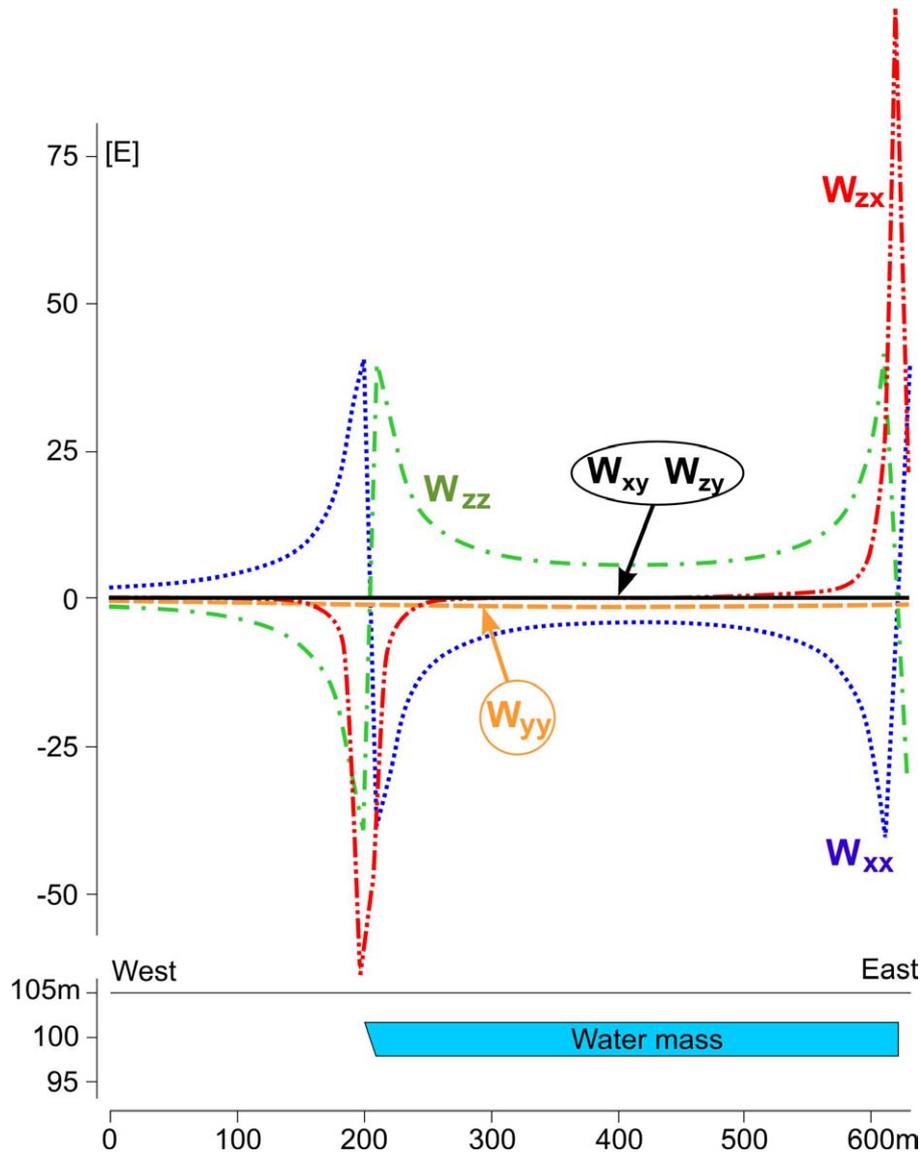


Fig. 4. The full gravitational tensor elements W_{xx} , W_{yy} , W_{zz} , W_{xy} , W_{zx} , W_{zy}

2. COMPUTATION OF GRAVITATIONAL CHANGES FROM GRAVITY GRADIENTS

Since certain relationships between various gravitational field functionals exist, it is sometimes useful to utilize these for conversion of our data or measurements. The advantage of using such formulas is that we do not need to have a physical (density) model of the area. That is, this computation is applicable in lack of a geological model or in those cases when our model is unreliable.

In this part of the paper we will focus on computing gravitational changes from certain combinations of W_{zx} , W_{zy} , W_{Δ} , W_{xy} tensor elements. The idea of the following computations is described in (VASSILIOU, 1986), extended by ideas found in (TÓTH, 2003).

In our first approach, the planar approximation of the z -component of the gravitational vector

g_z , can be computed by plane integration of W_{zx} , W_{zy} data over the area A (VASSILIOU, 1986)

$$g_z^{(1)}(x_P, y_P) = \frac{1}{2\pi} \iint_A \left[\begin{array}{l} W_{xz}(x, y) \frac{x_P - x}{(x_P - x)^2 + (y_P - y)^2} + \\ W_{yz}(x, y) \frac{y_P - y}{(x_P - x)^2 + (y_P - y)^2} \end{array} \right] dx dy. \quad (1)$$

It is also possible to compute the same quantity from $W_\Delta = W_{yy} - W_{xx}$, W_{xy} data by using the following planar version of the solution of the Eötvös gradiometric boundary value problem (TÓTH, 2003).

$$g_z^{(2)}(x_P, y_P) = \frac{1}{2\pi} \iint_A \left[\begin{array}{l} W_\Delta(x, y) \frac{(x_P - x)^2 - (y_P - y)^2}{[(x_P - x)^2 + (y_P - y)^2]^{3/2}} + \\ 2W_{xy}(x, y) \frac{2(x_P - x)(y_P - y)}{[(x_P - x)^2 + (y_P - y)^2]^{3/2}} \end{array} \right] dx dy \quad (2)$$

As a third choice, the above expressions for g_z can be combined with arbitrary weights that sum up to unity, i.e.

$$g_z = w g_z^{(1)} + (1 - w) g_z^{(2)} \quad \text{with } 0 \leq w \leq 1.$$

With the density model of the water mass it was possible to compute all the necessary gravity gradients from the above formulas (1-2). The deviations are modest (below 20 μGal) for formula (1), as it can be seen in the central cross sectional part of our model (see Fig. 5), whereas the g_z values computed by equation (2) deviate significantly from the model, especially over the central part of the Danube river bed. These deviations may come from the restricted model area, since the $W_\Delta = W_{yy} - W_{xx}$, W_{xy} model data have big values near the boundary of the area. With extended integration area the deviations should decrease significantly.

Several tests have been performed by restricting the integration up to a certain prescribed radius. One such computation is shown for the formula (1) when the integration radius of the gravity gradient data was only 100 m. The agreement with the model in this case is much worse, again especially bad over the central area of the model. The advantage of measuring the gravity gradients instead of gravity lies in the fact, that the possible site movements do not affect significantly the measured gravity gradients, whereas for gravity 10 cm site movement induces an approximately 31 μGal signal.

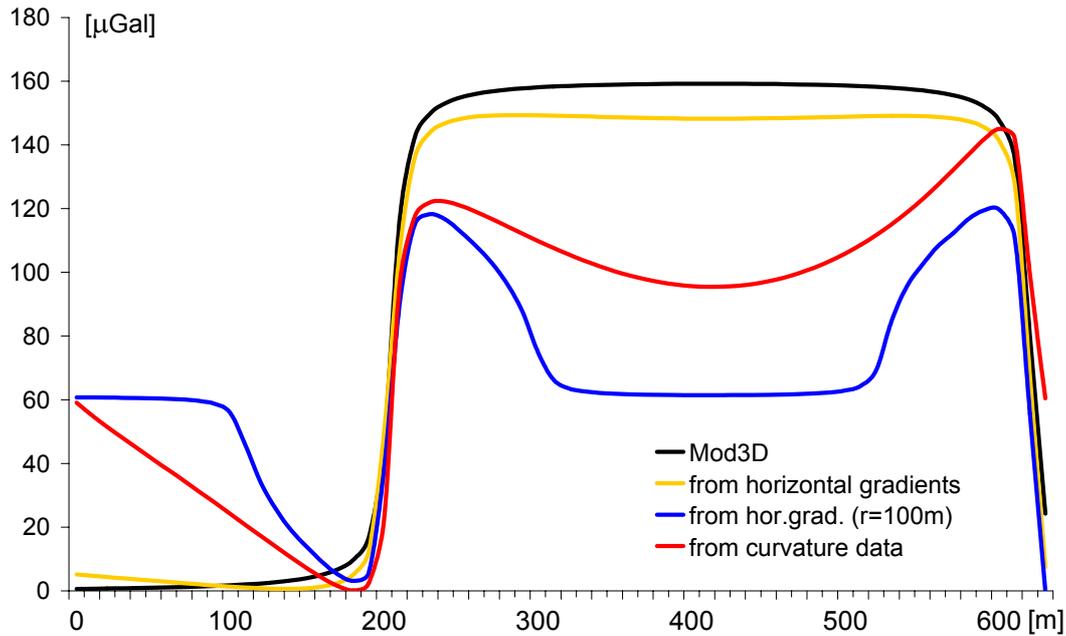


Fig. 5. The z-component of the gravitational vector

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