

CHANGES OF GRAVITY INFLUENCED BY WATER-LEVEL FLUCTUATIONS BASED ON MEASUREMENTS AND MODEL COMPUTATIONS

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ABSTRACT

More than 70% of the Earth's surface is covered by water. The outward form of this enormous volume of water is extremely variable. The majority is represented by oceans, seas and rivers but the quantity represented by subterranean water, arctic ice and precipitation (rain, snow) as well as that accumulated by human activity in reservoirs is not neglectable either. The influence of these volumes of water on the gravity field can be grouped in several ways: according to areal extension (global, regional, local), according to the causes producing the effects (tide, geodynamical processes, human activity), or according to the temporal variation of the causes (periodic, secular).

In our paper we are presenting the results of our gravity measurements investigating the influence of local water-table variations, the daily variation of the quantity of water in an 80000 m³ urban reservoir, and the water-level fluctuations of the Danube.

For the field work two LCR–G gravimeters were used, engineering geophysical soundings were carried out (using a penetrometer), and the available water-level observations of the region were analyzed. For modeling we have computed the gravity effects either by supposing a Bouguer plate or by decomposing the irregular body into a series of regular bodies. The effect of water masses on the topographic correction was also modeled.

The results can be useful in selecting localities for stations of a gravity base network or for studying secular variations of the gravity field by repeated measurements.

Keywords: changes of gravity, water-level change, density, engineering geophysical sounding, topographic correction

1. SEASONAL VARIATIONS OF WATER-TABLE AND MOISTURE CONTENT OF SOIL

The seasonal and long-time variation of gravity depends on the mass of ground water in the neighbourhood of the observation point. The increasing or decreasing level of the ground water is influencing the density of the nearsurface layers. The gravity effect of

this phenomenon can be modelled by a Bouguer plate because the difference between depth to the water-table and its areal extension exceeds several orders of magnitude. The gravity effect of a Bouguer plate is the following:

$$\delta g_B = 2\pi k \varrho H \quad (1)$$

where k is the gravity constant, ϱ is the density and H marks the thickness of the Bouguer plate, i.e. the variation of the water-table.

The effect of water-table variations on gravity was investigated in Hungary by one of the present authors (Szabó 1977). According to his theoretical calculations the effect may reach a maximum of 50 μGal . In the major part of Hungary the water-table can be found in young sediments, namely Holocene and Pleistocene sand, loess, alluvial gravel and clay except on the basin margins where older rocks crop out.

We have carried out our investigation in the Debrecen area (North-east Hungary). Under the western edge of the town there is a dividing line of N–S direction: to the east of it the surface consists of Pleistocene wind-blown sand, while to the west, loess is to be found on the surface. Depth to the water-table is 2–3 m where sand covers the surface, and 8–10 m under the loess covered area. According to (Rónai 1956) the water-table shows a daily, a seasonal and a decade-long period of variation. On summer days it deepens because of strong evaporation and during nights it rises. At the beginning of summer it starts to deepen, reaching its lowest level in the early autumn. After that it starts to rise to reach the highest level in late spring. On top of these two, Rónai and his colleagues found a 14–16 year long period as well. Many researchers studied the connection between precipitation and water-table. It was found that down-seeping water never reaches the water-table, even reaching 1 m depth occurs very rarely. Most of this water is used by the vegetation and the rest is off into the air by evaporation.

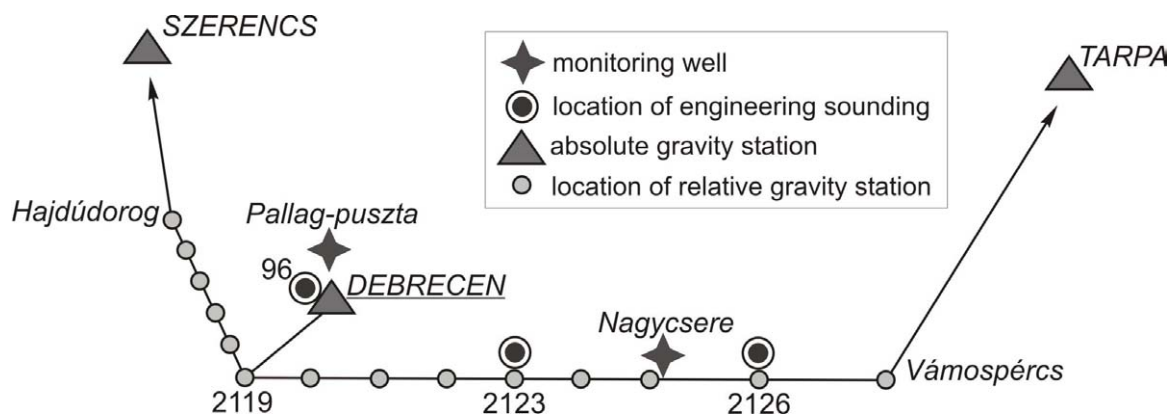


Fig. 1. Location sketch of measurements near Debrecen

The Debrecen area was selected for our investigations because we have the longest series of observation there. In 1989 an experimental line was located (see *Fig. 1*) to study the long-time variation of the gravity field (Csapó 1995). Along this line repeat measurements were carried out in regular intervals and even an absolute gravity station

was located there. The first absolute measurement was carried out in 1996, repeated in 2001 using an IMG C and an JILAG absolute gravity meter, respectively (Csapó 2002).

To compute the gravity effect of water-table variation we need two data, namely the extent of this variation and the volume difference between natural moisture content and saturation of layers containing ground water. Information on water-table variation is provided by registrations of monitoring wells. In the Debrecen area we have the data of two such wells, one about 4–5 km to the north-east of the town, at Pallag-puszta, the other one to the east of the town at the village of Nagycsere. From both wells the monthly averages are provided.

The Pallag-puszta well has an observation series between 1992 and 2002, with maximum difference of 1.47 m. The average depth to the water-table is about 8 m, in harmony with the generally found 8–10 m, characteristic for the area covered by loess.

To determine the bulk-density variation in connection with water-table variation is a more difficult task. Laboratory bulk-density measurements of loose sediments is a laborious and troublesome job. There is, however, a method, engineering geophysical sounding, which provides in situ data on several physical parameters of the penetrated layers, among them bulk density. This method is suitable for investigating near-surface unconsolidated formations which are penetrable by small-diameter pipes (0.042–0.052 m) pressed down by a hydraulic equipment, up to a maximum penetration of 20–30 m or till the first hard layer (limestone, marl, sandstone, etc.). When applying this method, all layers thicker than 0.2 m and of physical parameters differing from their neighbourhood can be detected. The physical parameters are registered in 0.1 m intervals either during pressing the pipe downwards or – after reaching the bottom – inside the pipe with different sondes.

The following parameters are registered:

- *Total pressure*, meaning the oil pressure in the hydraulic system, necessary for penetrating the layer and to overcome the friction on the wall of the pipe;
- *Cone pressure*, appearing at the cone of the pipe while it proceeds downwards into the layer. This pressure is proportional to the breaking strain of the layer;
- *Natural gamma activity*, measured within the pipe by a nuclear probe consisting of a KJ crystal and a photoelectron multiplier. This quantity is proportional to the potassium content (K^{40} isotope) of clays. As in these near-surface layers very rarely occur any other radioactive elements, the natural gamma activity indicates clay content;
- *Gamma-gamma intensity*, i.e. the scattering of the gamma particles coming from a screened artificial radiation source fitted to the end of the nuclear probe. Thus gamma particles do not reach the detector directly but by scattering in the layers surrounding the sonde (Compton effect). Scattering is proportional to the density of the medium the gamma particles penetrate, thus registered data can be converted into bulk density by high accuracy (deviation less than 1%). Conversion factors are determined by calibration in man-made standards of known parameters. The density jump unambiguously indicates the location of the water-table. By integrated interpretation of all registered parameters we can distinguish formations and determine beside bulk density the extent of water saturation. The small-diameter borehole remaining after the removal of the pipe rarely collapses immediately, thus the depth to the water-table can be measured directly.

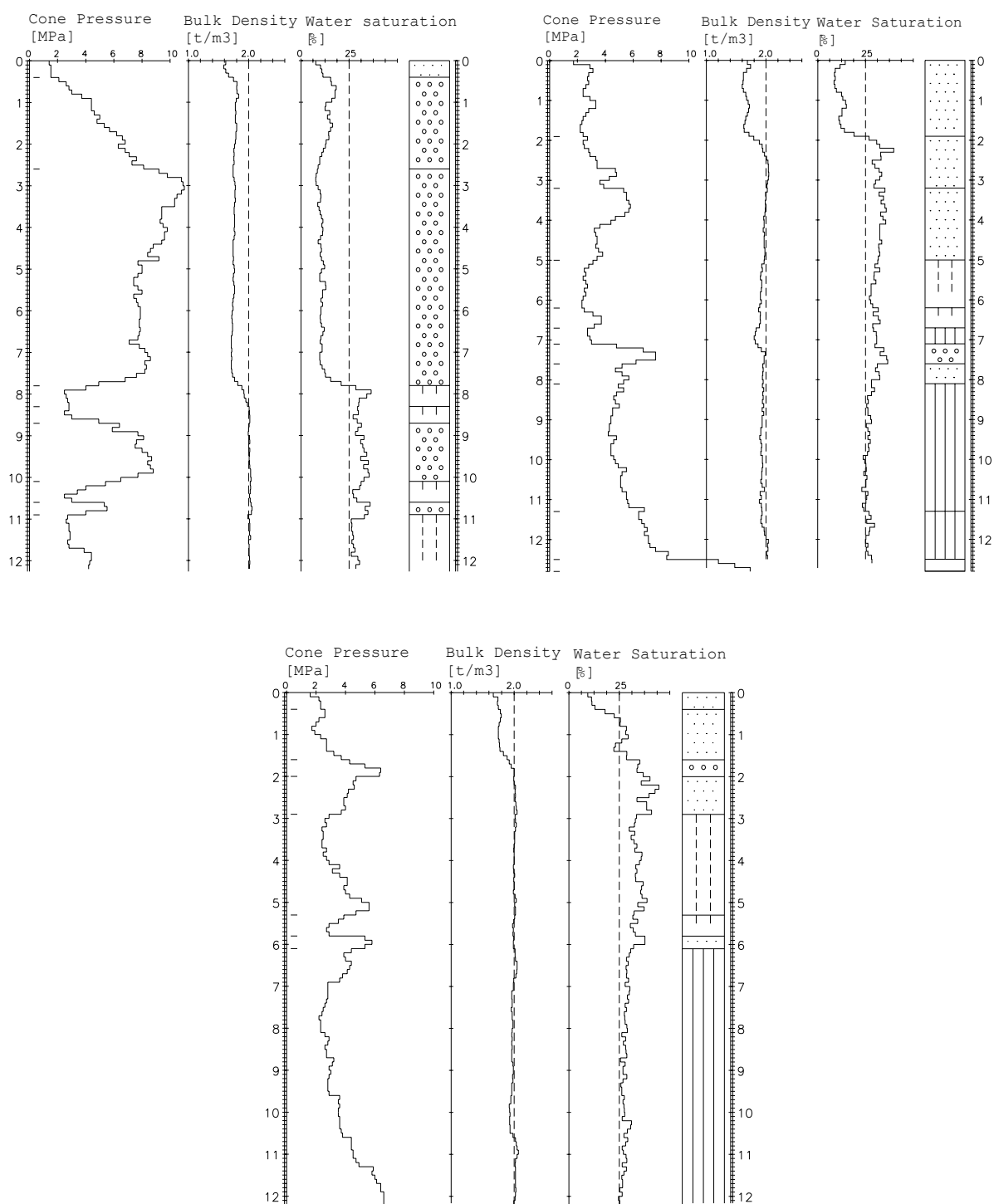


Fig. 2. Engineering geophysical sounding curves (stations 96, 2123 and 2126) (Cone pressure [Mpa], Bulk density [t/m³], Water saturation [%], Geological log)

On our request, ELGOSCAR International Hungarian-American Environmental Protection and Engineering Geophysical Ltd. carried out engineering geophysical soundings in the neighbourhood of the absolute gravity station of Debrecen and on two points of the experimental line for monitoring long-time variation of the gravity field. According to the sounding at the absolute gravity station (No. 96) ground water causes a

bulk-density increase of $\Delta\rho = 283 \text{ kg/m}^3$ (Fig. 2). In the near-by Pallag-pusztá monitoring well the maximum difference in the monthly average depths to water-table was found to be 1.47 m and the computed g value was 17.4 μGal . That means 11.9 μGal gravity effect for 1 metre ground water-level variation.

From this computation it follows that the g value of the Debrecen absolute gravity station – depending on the actual depth to water-table – may change 15–20 μGal .

The above results call attention to the fact that gravity changes due to water-table fluctuations may reach or even exceed the accuracy of gravity measurements. Accordingly, in relative measurements with the aim of indicating vertical motion, or in absolute measurements the depth to local water-table has to be registered as well. *However, the engineering geophysical soundings provide data fast and relatively cheaply as far as we know, they were never used for this purpose.*

The data set of the Nagycsere monitoring well contains a 20-year series of observations (1982–2002), with an average of 2.70 m depth to the water-table and a maximum fluctuation between 1.62 and 4.35 m. This well is to be found on the gravity profile established to monitor the subsidence of the Debrecen area. Three engineering geophysical soundings were located on three points of this profile, 10 km apart. These soundings provided significant bulk density changes, $\Delta\rho = 0.375$ and 0.250 t/m^3 , respectively, at the water-table (see Fig. 2). The respective gravity effects in g are 42.7 μGal (15.7 $\mu\text{Gal/m}$, at point 2123) and 28.6 μGal (10.5 $\mu\text{Gal/m}$, at point 2126).

Accordingly, the 0.125 t/m^3 difference in the bulk density change results in a 5.2 μGal gravity effect for one metre of water-table variation. Regarding the 2.72 m maximum variation in the water-table in the Nagycsere monitoring well, the extent of gravity effect caused by this fluctuation may reach 14.1 μGal between the two points of the profile.

The above data call our attention to the uncertainty of 10–15 μGal even in the results of one gravimetric link – caused by water-table fluctuations. This means that in the case of high-precision gravity measurements (absolute gravity observation, monitoring time-variation, etc.) we have to take into account the gravity effect of water-table fluctuations.

2. INVESTIGATION OF GRAVITY EFFECT OF WATER VOLUME FLUCTUATION IN RESERVOIRS

The gravity effect of the daily water-level fluctuation of the Gellért hill reservoir of Budapest (capacity: 80000 m^3) was studied by relative gravity measurements. One of the gravity stations was located above the reservoir (the concrete ceiling is covered by 3–4 m of gravel and soil), the other station was at a distance of 500 m, where the water of the reservoir had already no gravitational effect (see Fig. 3).

The observations were repeated several times with two gravimeters in the morning and in the afternoon. The volumes of water at the time of measurements were computed using the dimensions of the reservoir and the water-level registration of the Metropolitan Waterworks. The observed Δg values are presented in Table 1, where Δg

means the observed relative gravity (in μGal) for the given gravimeter and H means the height (in m) of water in the reservoir. The change per metre was calculated from the actual $\delta\Delta g$ and ΔH data, where $\delta\Delta g$ means the gravity variation due to the daily water-level change. One metre of water-level change corresponds to 10000 m^3 of water.

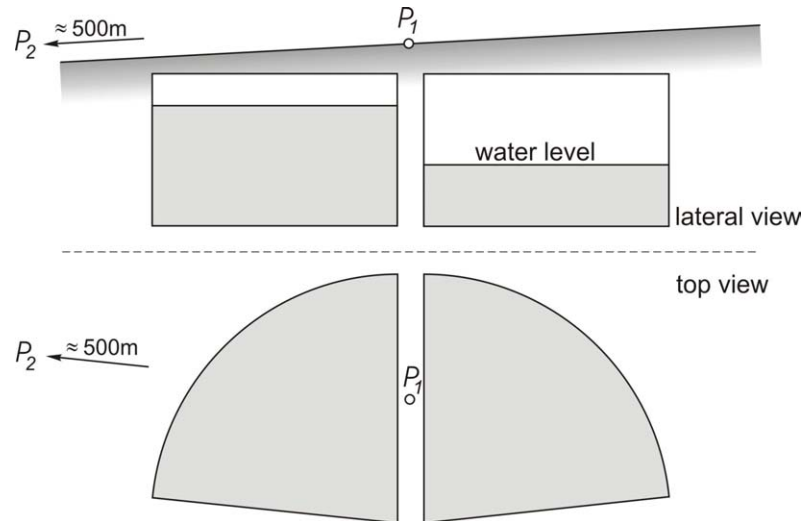


Fig. 3. Measurement set-up and modelling sketch of the Gellért hill reservoir

The theoretical effect was calculated by decomposing the causative body into rectangular prisms (HAÁZ 1953). The method is based on the fact that the gravity effect of any irregular body can be calculated by decomposing it into regular bodies whose effect can be calculated easily, and then summing their effects. The result of this computation provided a value of $32 \mu\text{Gal/m}$. The difference between measured ($27.2 \mu\text{Gal/m}$) and computed values may be due to the uncertainty in the relative position of the observation point and the reservoir.

Table 1. Gravimeter measurements above the Gellért-hill water reservoir

| day | part | LCR 1919-G | | | | | LCR 963G | | | | |
|------------|------|------------------------|------------------|------|------------|-----------------------------|------------|------------------|------|------------|-----------------------------|
| | | Δg | $\delta\Delta g$ | H | ΔH | $\delta\Delta g / \Delta H$ | Δg | $\delta\Delta g$ | H | ΔH | $\delta\Delta g / \Delta H$ |
| 1. | a.m. | 477 | 126 | 7.35 | 4.82 | 26.1 | 475 | 116 | 7.11 | 4.59 | 25.5 |
| | p.m. | 351 | | 2.53 | | | 359 | | 2.52 | | |
| 2. | a.m. | 468 | 92 | 7.29 | 3.71 | 24.8 | 475 | 122 | 6.94 | 3.55 | 34.4 |
| | p.m. | 376 | | 3.58 | | | 353 | | 3.39 | | |
| 3. | a.m. | 484 | 102 | 7.42 | 3.60 | 28.3 | 477 | 93 | 7.22 | 3.91 | 23.8 |
| | p.m. | 382 | | 3.82 | | | 384 | | 3.31 | | |
| mean (1-3) | | | | | | 26.4 | | | | | 27.9 |
| average | | 27.2 $\mu\text{Gal/m}$ | | | | | | | | | |

These results contain a warning: we should not locate gravity stations at places where the movement of large volumes of water may be presumed. The repeat measurements of such links may show deviations of 20–30 μGal , exceeding the reliability level of present-day relative gravimeter measurements.

3. INFLUENCE OF WATER-LEVEL FLUCTUATIONS OF THE DANUBE ON THE GRAVITY FIELD

The measurements were carried out during the summer flood of 2002, with links similar to those described in the previous chapter. One station was located on top of the right-hand embankment at Szabadság-bridge, the other one about 500 m to the west, at the foot of Gellért hill. Although the measurement set-ups are similar, the gravity effects of the two situations are rather different. In the former case the total effect appears in the station above the reservoir (the water-level fluctuations are possible to indicate even with a single registration gravimeter). In the present case one of the gravity stations is influenced not only by the level of the Danube, but by ground-water fluctuations as well.

Measurements were carried out on four subsequent days because of the fast recession of the flood. Daily observations were carried out in a P1–P2–P1–P2–P1–P2 sequence. The averages of the daily 5 Δg values for both gravimeters are presented for the first and the last day, in *Table 2*. The maximum subsidence of the level of the Danube was 4.16 m and the respective gravity change 41 μGal .

Table 2. Gravimeter measurements at the Danube embankment

| day | water-level | LCR-1919 | LCR-963 | mean Δg | $m_{\Delta g}$ |
|----------|-------------|----------|---------|-----------------|----------------|
| | (m) | | | | |
| 1. | 0,00 | 696 | 679 | 688 | ± 12 |
| 4. | - 4,16 | 653 | 640 | 646 | ± 14 |
| Δ | 4,16 | 43 | 39 | 41 | |

To verify the measurements, model calculations were carried out (see *Fig. 4*).

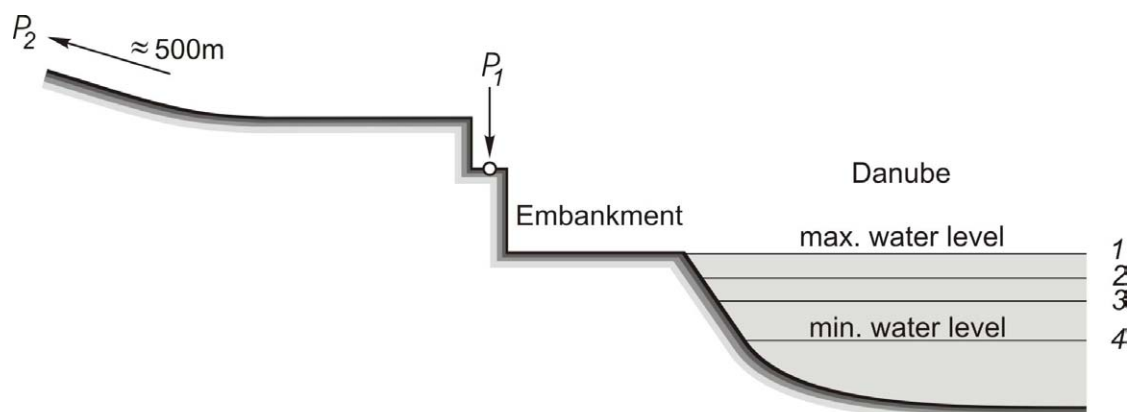


Fig. 4. Sketch for modelling the Danube flood

The variation of the Danube water was approximated by horizontal prisms (RASMUSSEN-PEDERSEN, 1979). The total effect of the 4.16 m water-level variation was found to be 21.7 μGal , about half of the measured one. We suppose that the difference is due to the water-table fluctuation resulting from the water-level variation of the Danube. If the water-table follows the water-level of the Danube without any phase lag,

to substitute the missing $\sim 20 \mu\text{Gal}$ gravity effect a permeability of about 14% is necessary. This value seems realistic for the alternating argillaceous marl and sand complex forming the river bank.

4. THE EFFECT OF WATER-LEVEL FLUCTUATIONS OF THE DANUBE ON THE TOPOGRAPHIC CORRECTION

We have calculated the effect of the flood on the topographic correction by approximating the surface of the water and the surrounding topography by a three-dimensional digital terrain model of $10 \times 10 \text{ m}$ grid spacing (*Fig. 5*).

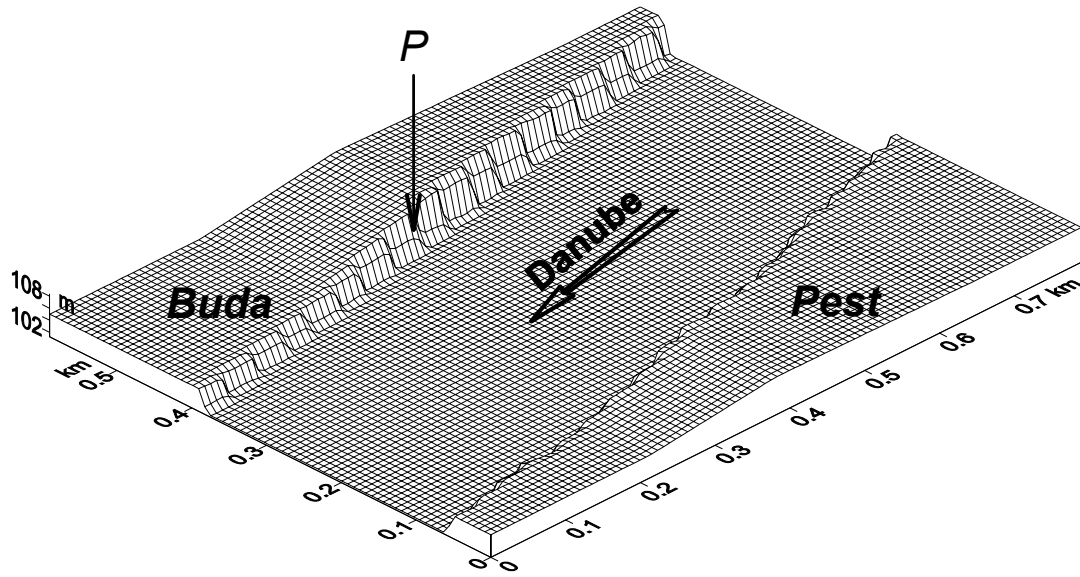


Fig. 5. Digital terrain model

The topographic attraction can be computed as the sum of the effects of vertical mass prisms, whose mass can be approximated by vertical „mass lines” which concentrate the masses of the prisms. The sum of their attraction can be efficiently computed via the two-dimensional fast Fourier transform (Li 1993, Mnyerczán and Szócs 2002, Rózsa 2002).

Starting from the well-known Heiskanen–Moritz formula [1967], the value of δg_{ij} terrain correction in point (x_i, y_j) of height h_{ij} is:

$$\delta g_{ij} = -f \int_x \int_y \int_z \frac{\rho(x, y, z) \cdot (h_{ij} - z)}{l^3} dx dy dz \quad (2)$$

where

$$l = \sqrt{(x_i - x)^2 + (y_j - y)^2 + (h_{ij} - z)^2}$$

and f the gravity constant. If the digital topography is modelled by $N \times M$ vertical prisms and supposing a constant \mathcal{G} density, the above integral can be written in the following form:

$$\delta g_{ij} = -f\mathcal{G} \sum_{n=1}^N \sum_{m=1}^M \int_{x_n - \Delta x/2}^{x_n + \Delta x/2} \int_{y_m - \Delta y/2}^{y_m + \Delta y/2} \int_{h_{ij}}^{h_{nm}} \frac{(h_{ij} - z)}{l^3} dx dy dz$$

After integration we get:

$$\delta g_{ij} = -f\mathcal{G} \Delta x \Delta y \sum_{n=1}^N \sum_{m=1}^M \left[x \ln(y+l) + y \ln(x+l) - z \arctan \frac{xy}{zl} \right]_{x_i - \frac{\Delta x}{2}}^{x_i + \frac{\Delta x}{2}} \left|_{y_j - \frac{\Delta y}{2}}^{y_j + \frac{\Delta y}{2}} \right|_{h_{ij} - h_{nm}}^0 \quad (3)$$

where Δx and Δy are the grid spacing.

Evaluation of this relation needs extremely much computer time. It is more economical to use multiplication instead of convolution by transforming both the topographic model and the filter by Fourier transform. After multiplication we get the required effect by inverse Fourier transform. The method was elaborated by (Li 1993) together with the necessary computer program. With this method we get the effect of the water mass not only in the point of measurement (P), but in its whole environment.

As topographic correction depends not only on the disturbing masses but also the height of the point of measurement, it is not enough to know the water-level fluctuations but the whole topographic model is necessary for the computations. Therefore we have determined the topography in a 800×600 m environment of the point of measurement in a $\Delta x = \Delta y = 10$ m grid.

As the purpose of our computations was the determination of the changing topographic correction due to the flood, it can be done by forming the difference of two attractions, i.e. by subtracting the effect of the normal water-level from that of the highest level. This computation was carried out for each grid point of the terrain model. Accordingly, the effect of the masses along the river banks are cancelled out but we get the effect of the flood in the real height where the measurements could have been carried out. It has to be emphasised that over the river, the „surface of measurements” is different in the two cases, in the first one it is the normal water-level while in the second one, the highest level of the flood.

In *Fig. 6*, the difference of the two topographic corrections are presented in the neighbourhood of the observation point.

As can be seen, this difference has two maximums along the banks of the river decreasing rapidly in both directions. This effect is due to different causes: on the banks to the water mass of the flood, while over the river to the fact that the levels of reduction differ. In this latter case, however, appears the gravity effect of the river-side masses between the two water-levels, also decreasing rapidly with distance. The closely occurring local peaks are caused by both the not fine enough grid spacing and by the

grid not being parallel with the banks. These peaks can be removed by eliminating the above causes.

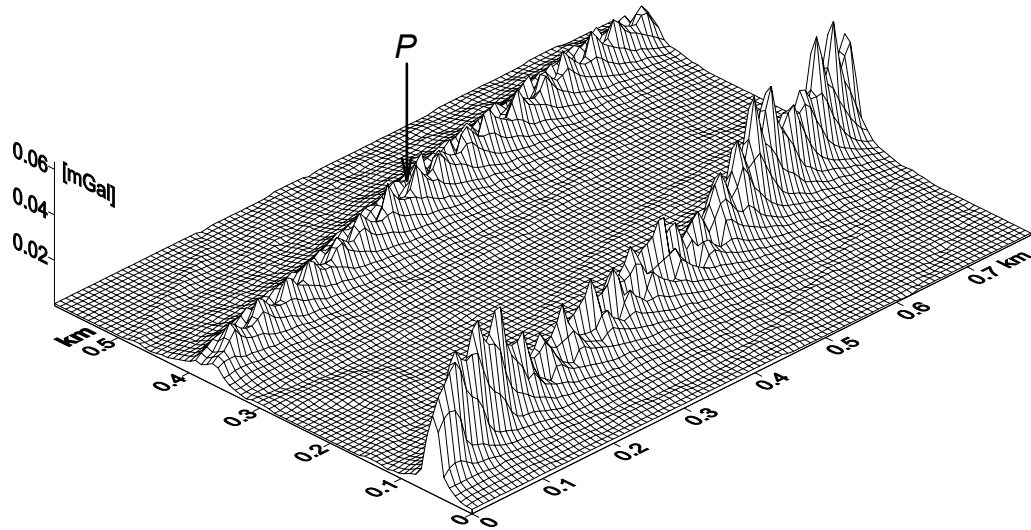


Fig. 6. Distribution of the difference between topographic corrections computed for the highest and lowest water-level of the Danube

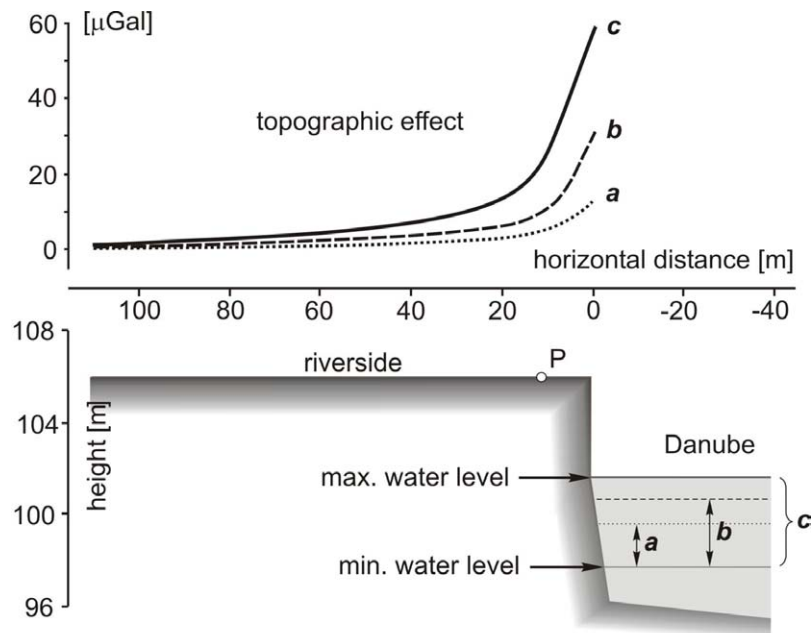


Fig. 7. Topographic effect versus horizontal distance for different water-levels

In Fig. 7, the gravity effects of the water masses versus distance are plotted in a section perpendicular to the river bank. In the lower part of the figure the section of the terrain model is presented with three water-levels ($a=1.12$ m, $b=2.08$ m, $c=4.16$ m), while in the upper part the respective gravity effects. As can be seen, if the flood is only 2 m high,

only 1–2 μGal variation can be registered at a distance of 40 m. But even the effect of a 4 m high flood cannot be detected over a distance of 80–100 m. At the same time, in the immediate neighbourhood of the river bank this effect can reach a value as high as 60 μGal . In point *P*, the difference of the gravity effect between lowest and highest water-level was found to be 22 μGal .

The measurements and model calculations prove that in the case of gravity base networks, locating stations near river banks one has to consider the gravity effect of water-level fluctuations.

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REFERENCES

- CSAPÓ G. (2000): The new gravity base network (MGH-2000) of Hungary (in Hungarian). *Geomatikai Közlemények III*, pp. 203–214. Sopron
- HAÁZ I. B. (1953): Relations between the potential of the attraction of the mass contained in a finite rectangular prism and its first and second derivatives (in Hungarian with English abstract). *Geophysical Transactions II*, 7, pp. 57–66
- HEISKANEN W. A. and MORITZ H. (1967): *Physical Geodesy*. Freeman & Co. San Francisco
- LI Y.C. (1993): *Optimized spectral geoid determination*. UCGE Reports No. 20050, Calgary
- MNYERCZÁN A., SZŐCS K. (2002): *A dunai árvíz hatása a nehézségi erőtérré*. Department of Geodesy, Budapest University of Technology and Economics
- RÓNAI A. (1956): *Das Grundwasser der ungarischen Becken, Ergebnisse der ungarischen Grundwasser Kartierung 1950–1955* (in Hungarian with German abstract). *MÁFI évkönyv XLVI.*, 1.
- RASMUSSEN-PEDERSEN (1979): *Geophysical Prospecting*, Vol.27. No.4, pp.
- RÓZSA SZ. (2002): *Application of field corrections of gravity measurements in high-precision geoid determination* (in Hungarian). (Ph.D. Thesis)
- SZABÓ Z. (1977): *Influence of water-table fluctuations and ratio of saturation on gravity measurements* (in Hungarian). *Magyar Geofizika XVIII*, 4, pp. 121–126

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