HIGH-RESOLUTION MEASUREMENTS OF NON-LINEAR SPATIAL DISTRIBUTION OF GRAVITY GRADIENTS IN HUNGARY

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Abstract

Linear changing of the gravity gradients between the adjoining network points is an important demand for different interpolation methods in geodesy (e.g. interpolation of the deflection of the vertical, geoid computations, and interpolation of the gravity values or the vertical gradients of gravity). To study the linearity of gravity gradients, torsion balance measurements were made both at the field and in a laboratory: one is at the southern part of the Csepel island, and the other in the Geodynamical Laboratory of Loránd Eötvös Geophysical Institute in the Mátyás cave. The results of our investigations show that the linearity of the gravity gradients mainly depends on the given point density and the the geological fine structure of rocks and shallow subsurface density. It seems the given point density of the earlier torsion balance stations may be not enough for some geodetic purposes, moreover the problem could not be solved even applying topographic reduction of gravity gradients.

Keywords. gravity gradients, curvature data, linearity, torsion balance

1 Introduction

In the last century thousands of torsion balance measurements were made for geophysical purposes in Hungary, and now these measurements are waiting for geodetic applications. During our former researches a suspicion was aroused about the nonlinearity of the gravity gradients between the neighboring torsion balance stations. The question is whether the point density of these measurements is enough or not to satisfy the linear changing requirements of gravity gradients which is an important demand of the geodetic applications?

First a simple example can be found here for the geodetic demand of the linearity and then some new test measurements and investigations are analyzed.

2 Necessity of the linearity analysis

If difference of deflection of the vertical wants to be determined by an interpolation method based on the curvature gradients $W_\Delta = W_{yy} - W_{xx}$ and W_{xy} measured by torsion balance in α_{ik} azimuth between the points P_i and P_k, the

$$\int_{n_i}^{n_k} \frac{\partial^2 W}{\partial n \partial s} dn \qquad (1)$$

integral (1) should be computed, where

$$\begin{aligned} \frac{\partial^2 W}{\partial n \partial s} &= \frac{1}{2}\left(\frac{\partial^2 W}{\partial y^2} - \frac{\partial^2 W}{\partial x^2}\right)\sin 2\alpha_{ik} + \frac{\partial^2 W}{\partial x \partial y}\cos 2\alpha_{ik} \\ &= \frac{1}{2} W_\Delta \sin 2\alpha_{ik} + W_{xy}\cos 2\alpha_{ik} \end{aligned} , \qquad (2)$$

n_{ik} is the distance between P_i and P_k, and s is the direction perpendicular to n (Völgyesi 2005).

If the P_i and P_k points are quite close to each other, then the change of the second derivative of the potential W_{ns}, can be considered as linear, and the integral (1) can be approximated with the following formula:

$$\int_{n_i}^{n_k} \frac{\partial^2 W}{\partial n \partial s} dn = \frac{1}{2}\left[\left(\frac{\partial^2 W}{\partial n \partial s}\right)_i + \left(\frac{\partial^2 W}{\partial n \partial s}\right)_k\right] n_{ik} \,. \qquad (3)$$

The change of the N–S and E–W components of deflection of the vertical $\Delta\xi_{ki}$ and $\Delta\eta_{ki}$ can be determined using the following simplification between the points P_i and P_k

$$\frac{1}{2}\left[(\Delta W_{ns})_i + (\Delta W_{ns})_k\right] n_{ik} = g\left(\Delta\xi_{ki} \sin\alpha_{ik} - \Delta\eta_{ki}\cos\alpha_{ik}\right) \qquad (4)$$

where

$$\Delta W_{ns} = \frac{1}{2}(W_\Delta - U_\Delta)\sin 2\alpha_{ik} + (W_{xy} - U_{xy})\cos 2\alpha_{ik} \;, \qquad (5)$$

the U_Δ and U_{xy} are the normal values of the W_Δ and W_{xy} curvature gradients measured by the torsion balance (Völgyesi 2005).

The accuracy of the interpolated deflection of vertical primarily depends on the linear or nonlinear changing of the curvature gradients W_Δ and W_{xy} between the adjacent points.

The same problem occur in case of the interpolation of gravity g, gravity anomaly Δg or vertical gradient W_{zz} based on the horizontal and curvature gradients of gravity measured by torsion balance, the base principle of these method is quite similar to the interpolation of the deflection of the vertical as discussed above (Völgyesi 2005, Völgyesi et al 2005, Völgyesi et al 2007, Dobróka and Völgyesi 2008, Tóth and Merényi 2005, Tóth and Völgyesi 2005).

3 The test area

In the last century approximately 60000 torsion balance measurements were made in Hungary. Among others on the Csepel island 238 measurements were made in the year 1950 for geophysical purposes. For the linearity test 7 torsion balance points was selected from these 238 measurements at the southern part of the island. The numbers of selected points are: E220, E218, E238, E208, E206, E204, E207 as it can be seen on Fig. 1. The average distances between these points are about 1.5 km. The test area had formerly been flooded by the river Danube with only a few meters topographic relief (Fig. 2). To study the linearity of gravity gradients new torsion balance measurements 3.a – 3.b – 3.c – 3.d – 3.e were made too with higher point density between the points E238 and E208 (distances between these new points are 150 m) .

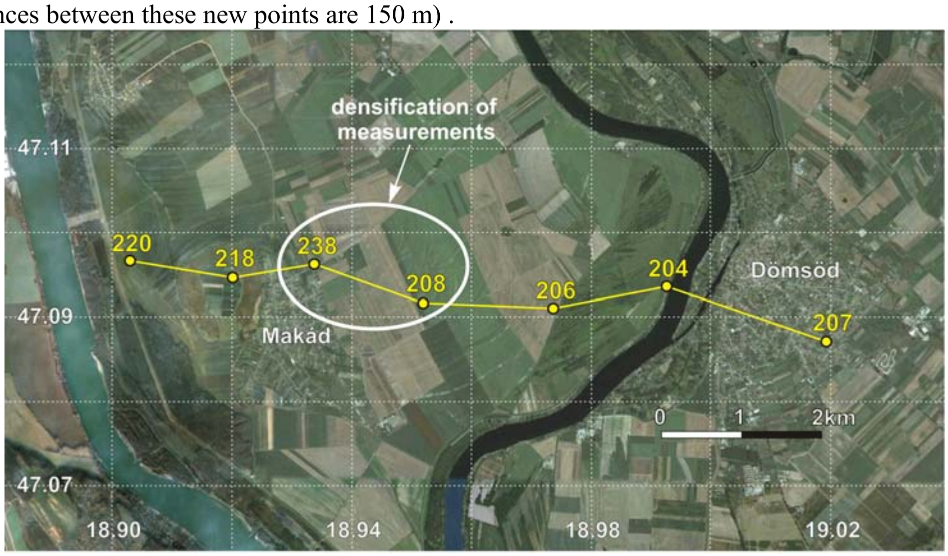

Fig. 1. Torsion balance points in the southern part of the Csepel Island

Fig. 2. New torsion balance measurement near the point E208

The other test area can be found in the Geodynamical Laboratory of Lorand Eötvös Geophysical Institute in the Mátyás cave. The location plan of the Geodynamical Laboratory can be seen on Fig. 3. The Hungarian gravity basepoint marked by the No. 82 can be found in the biggest hall of the cave, and the 14 gravity microbase network points are in the passageway between the entrance and the gravity basepoint. The distances between the microbase network points are only a few meters. Gravity values and the elements of the full Eötvös tensor are known on each point (Csapó 1991). There are unusually large gravity gradients along the network line because of the tall bluffy cliff at the entrance of the cave.

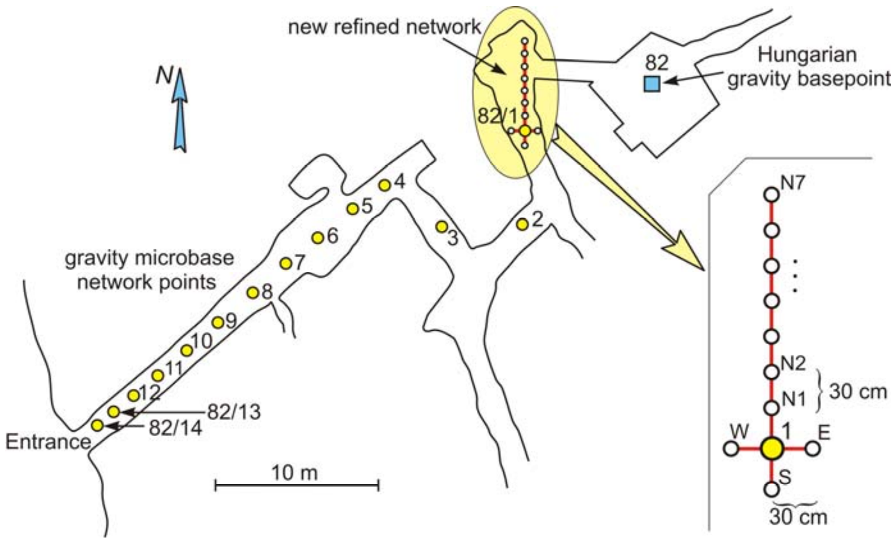

Fig. 3. The torsion balance microbase and the refined network points in the Mátyás cave

New torsion balance measurements were made on the microbase points in 2008-2009 by a refurbished AUTERBAL instrument (Völgyesi and Ultmann 2012). Later to study the fine structure of the gravity gradients the torsion balance stations are refined around the point 82/1, in N-S and E-W direction (see on Fig. 3). This ultra-fine network contains 9 points in N-S and 3 points in E-W direction, including the point 82/1. Distances between points are 30 cm.

The Mátyás cave is particularly good place for precise measurements because of the constant temperature and low microseismic noise.

For all the gravity gradients measured by torsion balance 1E (1E = 1 Eötvös Unit = 10^{-9} $1/s^2$) standard deviations have been computed or estimated (Mueller et al 1968). To control this data repeated measurements were performed on the Hungarian gravity basepoint 82 (see on Fig. 3) by our AUTERBAL torsion balance. Results of our measurements on the gravity basepoint, averages and the standard deviations (*SD*) of the gravity gradients can be found in Tab. 1. A little bit worse *SD* values 2E for the W_{zx} ,W_{zy} , W_{xy} and 5E for the W_{Δ} can be derived from our measurements.

Standard deviations of the topographic reduction (between 0-100m) can be expected 3E (Mueller et al 1968).

Table 1. Results of the repeated torsion balance measurements on the gravity basepoint and the standard deviations *SD* of the measured gravity gradients. [E] is the Eötvös unit (1E=10^{-9} s^{-2})

	Wxz [E]	*Wyz* [E]	*W∆* [E]	*Wxy* [E]
1	-236.5	-176.4	-354.2	-116.6
2	-236.7	-175.4	-351.2	-117.3
3	-236.1	-174.2	-353.3	-116.8
4	-241.0	-173.7	-343.3	-120.2
5	-239.6	-171.3	-342.1	-122.4
6	-240.4	-171.1	-342.5	-119.1
7	-237.5	-171.3	-343.3	-120.3
Average	-238.3	-173.4	-347.1	-118.9
SD	**1.9**	**2.0**	**5.1**	**2.0**

4 Analysis of the measurements

4-1. Csepel island test area

First, the 7 points E220, E218, E238, E208, E206, E204, E207 measured in 1950 located more or less along a straight line were selected (see Fig. 1). All the horizontal and curvature gradients W_{zx}, W_{zy}, W_{Δ}, W_{xy} measured by torsion balance and all of these values corrected with topographic reduction were available at these 7 points. For the computation of the topographic reduction leveling were carried out at 8 directions around all torsion balance stations at the distances 1.5, 3, 5, 10, 20, 30, 40, 50, 70, 100 m, according to Selényi (1953). The test area is nearly flat with only a few meters undulation of topography as it can be seen on Fig. 2, so the gravity effects of the topographic masses are negligible beyond 100 meters.

Horizontal and curvature gradients marked by circles, measured by torsion balance in 1950, while all of these gradients corrected with topographic reduction marked by triangles can be seen on the upper part of Figs. 4 and 5. Distances between the points are depicted on the horizontal axes of the figures, the mean distance is about 1.5 km.

Based on the behavior of the curves a suspicion was raised about the nonlinearity of the gradients between the neighboring points and the variation of the gravity gradients may contain components with higher frequency. To check this suspicion, new torsion balance measurements were performed between the point E238 and E208 with smaller distances. The new measurements were made by the support of the OTKA project K60657 managed by G. Csapó between the year 2006 and 2010 (Csapó 2010, Csapó et al 2009a, 2009b). The distances between the new points are 150 m. The finer resolution pictures of the gradients based on the new more detailed measurements can be seen on the lower part of Figs. 4 and 5.

The value of R^2 of the linear regression was applied to check and characterize the linearity of the torsion balance measurements:

$$R^2 = 1 - \frac{SSE}{SST}, \tag{6}$$

where *SSE* is the residual sum of squares and *SST* is the total sum of squares (Miller and Wichern, 1977). The better the linear regression fits the data in comparison to the simple average, the closer the value of R^2 is to one. The R^2 values have been computed with various combinations for the torsion balance measurements and the results are summarized in Tabs. 2-7.

The first columns of all tables show the various combinations of neighboring points, first all the 3 neighboring points, then 4 points … and finally the whole line. In the second columns the length between the two terminal points of the given combination can be seen in [m], and in the remaining columns the computed R^2 and $\overline{R}^2$ values are shown for the different gravity gradients ($\overline{R}^2$ is the mean value of different R^2). In several combinations of points some bigger differences can be found between R^2 values. This may be caused by measurement errors, or local subsurface mass inhomogeneities next to the torsion balance stations. Further investigations are necessary to discuss these problems.

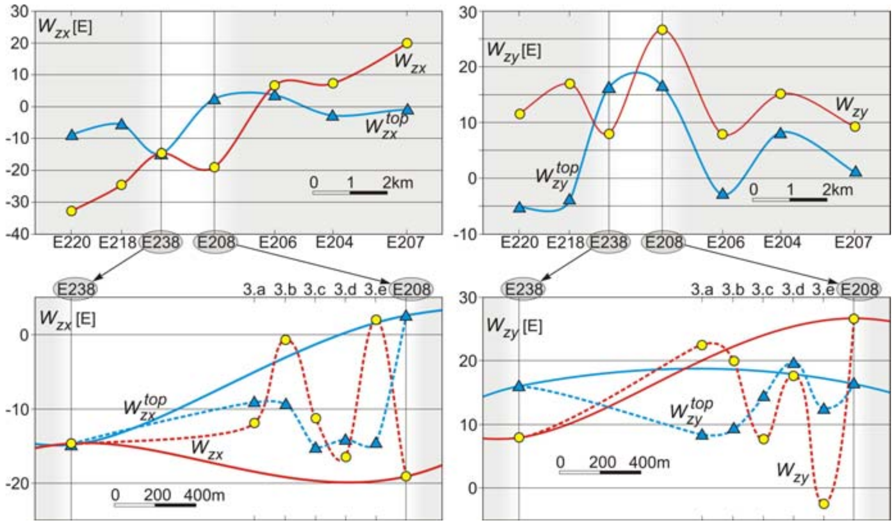

Fig. 4. Measured and corrected horizontal gravity gradients marked by circles and triangles respectively in the original and the refined network points on the Csepel island

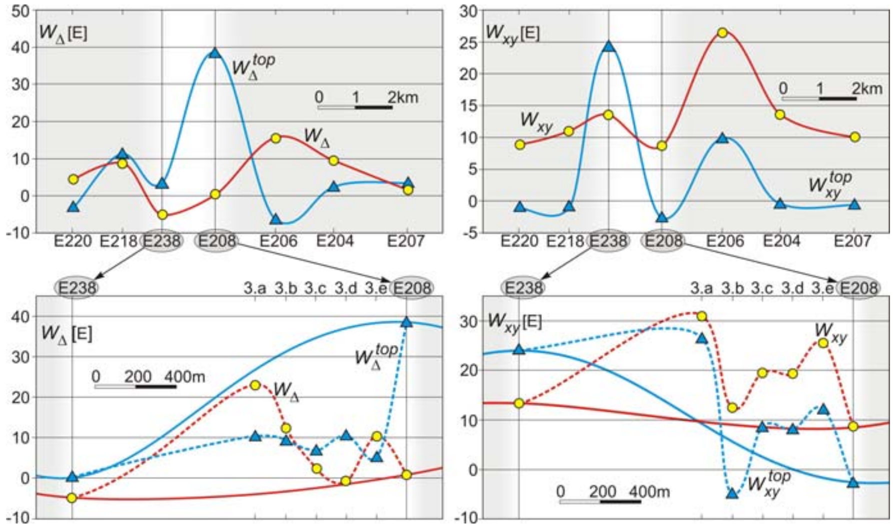

Fig. 5. Measured and corrected curvature data marked by circles and triangles respectively in the original and the refined network points on the Csepel island

Table 2. The R^2 values of the linear regression of the earlier torsion balance measurements on the line of the Makád test area in Csepel island without topographic reduction.

Points	Distance [m]	*Wzx* R^2	*Wzx* $\overline{R}^2$	*Wzy* R^2	*Wzy* $\overline{R}^2$	*WΔ* R^2	*WΔ* $\overline{R}^2$	*Wxy* R^2	*Wxy* $\overline{R}^2$
220-218-238	2386	0.99		0.12		0.40		0.99	
218-238-208	2503	0.24		0.35		0.27		0.28	
238-208-206	3083	0.64	0.70	0.00	0.17	0.95	0.60	0.53	0.55
208-206-204	3211	0.78		0.38		0.36		0.08	
206-204-207	3580	0.84		0.01		1.00		0.86	
220-218-238-208	3823	0.68		0.38		0.28		0.01	
218-238-208-206	4149	0.75	0.73	0.02	0.21	0.21	0.29	0.52	0.16
238-208-206-204	4648	0.74		0.00		0.68		0.10	
208-206-204-207	5226	0.85		0.45		0.01		0.03	
220-218-238-208-206	5470	0.84		0.00		0.13		0.55	
218-238-208-206-204	5713	0.83	0.84	0.00	0.01	0.24	0.17	0.18	0.25
238-208-206-204-207	6664	0.86		0.04		0.16		0.00	
220-218-238-208-206-204	7034	0.89	0.90	0.00	0.04	0.18	0.11	0.29	0.15
218-238-208-206-204-207	7729	0.91		0.08		0.03		0.01	
220-218-238-208-206-204-207	9050	0.93	0.93	0.02	0.02	0.03	0.03	0.06	0.06

Table 3. The R^2 values of the linear regression of the earlier torsion balance measurements on the line of the Makád test area in Csepel island with topographic reduction.

Points	Distance [m]	*Wzx_top* R^2	*Wzx_top* $\overline{R}^2$	*Wzy_top* R^2	*Wzy_top* $\overline{R}^2$	*WΔ_top* R^2	*WΔ_top* $\overline{R}^2$	*Wxy_top* R^2	*Wxy_top* $\overline{R}^2$
220-218-238	2386	0.40		0.75		0.25		0.70	
218-238-208	2503	0.30		0.69		0.63		0.02	
238-208-206	3083	0.76	0.49	0.77	0.50	0.06	0.46	0.25	0.34
208-206-204	3211	0.60		0.20		0.59		0.03	
206-204-207	3580	0.36		0.08		0.77		0.70	
220-218-238-208	3823	0.23		0.79		0.73		0.02	
218-238-208-206	4149	0.52	0.38	0.00	0.37	0.03	0.31	0.00	0.11
238-208-206-204	4648	0.30		0.40		0.11		0.40	
208-206-204-207	5226	0.47		0.29		0.37		0.01	
220-218-238-208-206	5470	0.48		0.08		0.02		0.06	
218-238-208-206-204	5713	0.27	0.32	0.00	0.19	0.08	0.07	0.05	0.17
238-208-206-204-207	6664	0.21		0.49		0.11		0.41	
220-218-238-208-206-204	7034	0.33	0.28	0.10	0.06	0.00	0.05	0.00	0.06
218-238-208-206-204-207	7729	0.23		0.03		0.10		0.11	
220-218-238-208-206-204-207	9050	0.31	0.31	0.02	0.02	0.01	0.01	0.02	0.02

Table 4. The R^2 values of the linear regression of the new torsion balance measurements between the earlier points 238 and 208 on the line of the Makád test area in Csepel island without topographic reduction.

Points	Distance [m]	*Wzx* R^2	*Wzx* $\overline{R}^2$	*Wzy* R^2	*Wzy* $\overline{R}^2$	*WΔ* R^2	*WΔ* $\overline{R}^2$	*Wxy* R^2	*Wxy* $\overline{R}^2$
238-3a-3b	909	0.60		0.90		0.73		0.09	
3a-3b-3c	300	0.00		0.86		1.00		0.38	
3b-3c-3d	300	0.96	0.41	0.04	0.43	0.92	0.63	0.73	0.47
3c-3d-3e	300	0.49		0.26		0.49		0.73	
3d-3e-208	300	0.02		0.11		0.01		0.42	
238-3a-3b-3c	1059	0.30		0.11		0.26		0.08	
3a-3b-3c-3d	450	0.21	0.13	0.30	0.27	0.96	0.32	0.22	0.36
3b-3c-3d-3e	450	0.00		0.53		0.04		0.89	
3c-3d-3e-208	450	0.01		0.16		0.02		0.26	
238-3a-3b-3c-3d	1209	0.03		0.14		0.06		0.06	
3a-3b-3c-3d-3e	600	0.06	0.08	0.64	0.26	0.44	0.22	0.01	0.02
3b-3c-3d-3e-208	600	0.16		0.00		0.17		0.00	
238-3a-3b-3c-3d-3e	1359	0.20	0.12	0.02	0.03	0.08	0.30	0.14	0.19
3a-3b-3c-3d-3e-208	750	0.05		0.03		0.52		0.23	
238-3a-3b-3c-3d-3e-208	1509	0.01	0.02	0.02	0.02	0.01	0.01	0.00	0.01

In Tabs. 2 and 3 the results of the computations are summarized for the 7 points of the earlier torsion balance measurements E220, E218, E238, E208, E206, E204, E207 with and without topographic reduction respectively. In Tabs. 4 and 5 the results for the new torsion balance measurements 3.a – 3.b – 3.c – 3.d – 3.e between the points E238 and E208 can be seen.

Based on the data of Tabs. 2 – 5 decreasing the length of the measuring line improves the linearity of gravity gradients (because increases the values of R^2). Comparing the data of Tabs. 2, 4 and 3, 5 shows that decreasing distances between the torsion balance points from 1000-1500m to 150-300m does not result significantly the improvement of linearity.

Finally it can be concluded that the mean point density of the earlier torsion balance measurements does not provide the linear changing of gravity gradients between the neighboring network points. Moreover based on the data of Tabs 4 and 5 the problem could not be solved even applying topographic reduction. Otherwise this is likely due to the area of the former Danube floodplains where the mass density of the subsurface soil is extremely diverse. Further investigations would be useful to explore the fine structure of the soil mass inhomogeneities near to the surface.

Table 5. The R^2 values of the linear regression of the new torsion balance measurements between the earlier points 238 and 208 on the line of the Makád test area in Csepel island with topographic reduction.

Points	Distance [m]	Wzx_top R^2	Wzx_top $\bar{R}^2$	Wzy_top R^2	Wzy_top $\bar{R}^2$	WΔ_top R^2	WΔ_top $\bar{R}^2$	Wxy_top R^2	Wxy_top $\bar{R}^2$
238-3a-3b	909	0.96		0.93		0.92		0.00	
3a-3b-3c	300	0.77		0.85		0.92		0.28	
3b-3c-3d	300	0.61	0.69	1.00	0.60	0.12	0.54	0.72	0.37
3c-3d-3e	300	0.35		0.07		0.09		0.84	
3d-3e-208	300	0.75		0.17		0.62		0.00	
238-3a-3b-3c	1059	0.11		0.26		0.56		0.00	
3a-3b-3c-3d	450	0.74	0.50	0.93	0.34	0.01	0.33	0.13	0.13
3b-3c-3d-3e	450	0.51		0.19		0.21		0.26	
3c-3d-3e-208	450	0.64		0.00		0.56		0.12	
238-3a-3b-3c-3d	1209	0.01		0.00		0.65		0.00	
3a-3b-3c-3d-3e	600	0.70	0.33	0.42	0.22	0.35	0.48	0.17	0.17
3b-3c-3d-3e-208	600	0.28		0.25		0.43		0.32	
238-3a-3b-3c-3d-3e	1359	0.00	0.06	0.00	0.24	0.21	0.27	0.02	0.05
3a-3b-3c-3d-3e-208	750	0.12		0.47		0.33		0.09	
238-3a-3b-3c-3d-3e-208	1509	0.18	0.18	0.02	0.02	0.31	0.31	0.01	0.01

Further conclusion can be suspected based on the R^2 data and the behavior of the curves on the lower part of Figs 4 and 5, namely not even the 150 m point density of the torsion balance measurements seems to be enough for the linear behavior of gravity gradients between the neighboring points, – although the amplitude of the higher frequency changing terms are smaller yet.

4-2. Mátyás cave test area

Further researches were carried out to study the optimal distance between the torsion balance stations where the changing of gravity gradients can be considered as linear. The place of these investigations was the Mátyás cave in Budapest. Torsion balance measurements were made in 1991 in the Geodynamical Laboratory of Lorand Eötvös Geophysical Institute in the Mátyás cave where the Hungarian gravity basepoint and further 14 gravity microbase network points can be found (Csapó 2010). Torsion balance measurements were repeated here in 2008, the picture of the horizontal and the curvature gradients can be seen on Fig. 6.

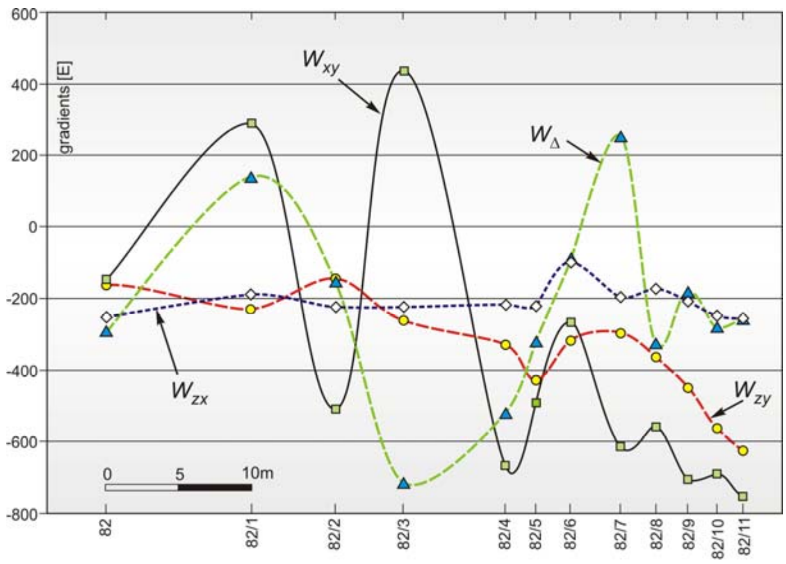

Fig. 6. Gravity gradients on the microbase network points in the Mátyás cave

Uncommon huge changing of gravity gradients were noticeable between points, the values can reach up to 1000 E (1E = 1 Eötvös Unit = 10^{-9} $1/s^2$) within a few meters. Based on the behavior of gradients on Fig. 6 and the R^2 data of Tab. 6 it can be established that the changing is not linear, the variation of the gravity gradients may contain further components with higher frequency. Further refined measurements were carried out to study the fine structure of the gravity gradients around the torsion balance station 82/1 in N-S and E-W direction. Distances between points are 30 cm (see on Fig. 3).

Table 6. The R^2 values of the linear regression of the torsion balance measurements on the line of the gravity microbase network points in Mátyás cave.

Points	Distance	*Wzx*		*Wzy*		*WΔ*		*Wxy*	
	[m]	R^2	$\overline{R}^2$	R^2	$\overline{R}^2$	R^2	$\overline{R}^2$	R^2	$\overline{R}^2$
4-5-6	4.4	0.79		0.01		1.00		1.00	
5-6-7	5.8	0.00		0.82		1.00		0.18	
6-7-8	5.8	0.66	0.58	0.29	0.68	0.09	0.55	0.70	0.58
7-8-9	4.6	0.09		0.99		0.54		0.35	
8-9-10	4.2	0.99		0.99		0.11		0.71	
9-10-11	3.8	0.92		0.98		0.56		0.54	
4-5-6-7	7.8	0.11		0.27		1.00		0.03	
5-6-7-8	8.2	0.00		0.23		0.04		0.25	
6-7-8-9	8.0	0.72	0.48	0.66	0.63	0.15	0.36	0.80	0.48
7-8-9-10	6.6	0.59		0.98		0.53		0.50	
8-9-10-11	6.0	0.97		0.99		0.07		0.80	
4-5-6-7-8	10.2	0.07		0.05		0.26		0.00	
5-6-7-8-9	10.4	0.05	0.43	0.03	0.46	0.01	0.25	0.49	0.49
6-7-8-9-10	10.2	0.83		0.78		0.26		0.77	
7-8-9-10-11	8.4	0.76		0.99		0.46		0.69	
4-5-6-7-8-9	12.4	0.00		0.09		0.17		0.08	
5-6-7-8-9-10	12.4	0.23	0.37	0.33	0.42	0.01	0.16	0.57	0.48
6-7-8-9-10-11	11.8	0.87		0.85		0.30		0.80	
4-5-6-7-8-9-10	14.4	0.06	0.22	0.38	0.45	0.07	0.05	0.17	0.41
5-6-7-8-9-10-11	14.2	0.38		0.53		0.03		0.66	
4-5-6-7-8-9-10-11	16.2	0.18	0.18	0.55	0.55	0.03	0.03	0.29	0.29

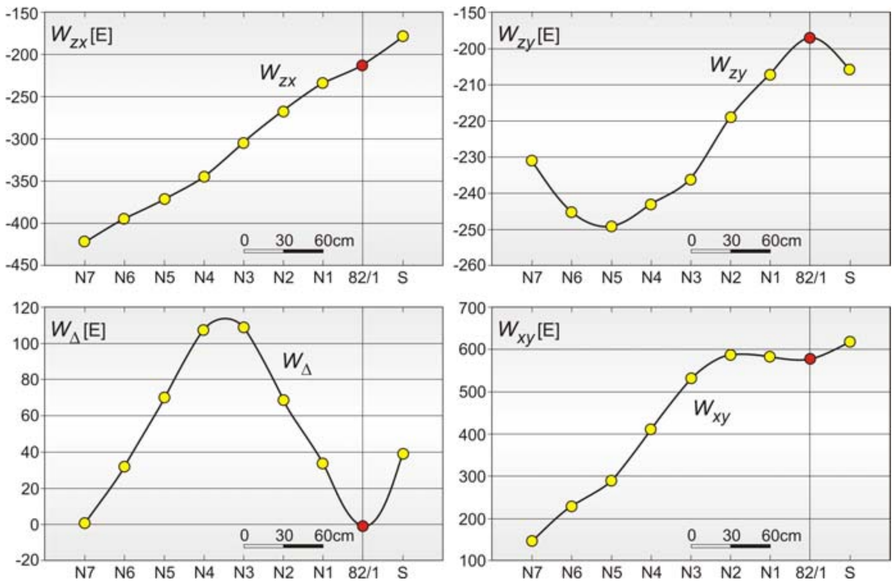

Fig. 7. Gravity gradients on the refined network in N-S direction in the Mátyás cave

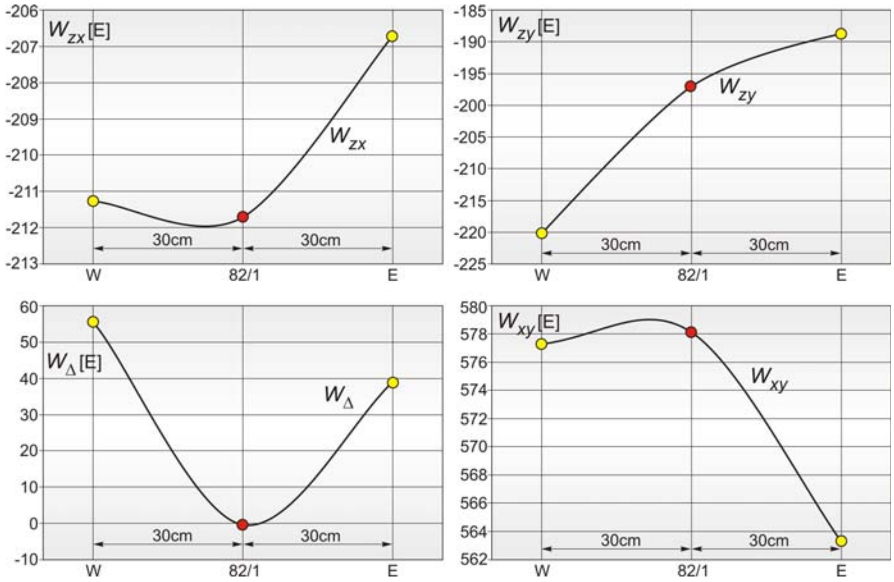

Fig. 8. Gravity gradients on the refined network in E-W direction in the Mátyás cave

Table 7. The R^2 values of the linear regression of the torsion balance measurements on the line of the refined gravity microbase points in Mátyás cave.

Points	Distance [m]	*Wzx* R^2	*Wzx* $\overline{R}^2$	*Wzy* R^2	*Wzy* $\overline{R}^2$	*WΔ* R^2	*WΔ* $\overline{R}^2$	*Wxy* R^2	*Wxy* $\overline{R}^2$
N7-N6-N5	0.6	1.00		0.90		1.00		0.99	
N6-N5-N4	0.6	1.00		0.11		1.00		0.96	
N5-N4-N3	0.6	0.98		1.00		0.78		1.00	
N4-N3-N2	0.6	1.00	0.99	0.94	0.71	0.72	0.79	0.96	0.89
N3-N2-N1	0.6	1.00		0.99		1.00		0.67	
N2-N1-82/1	0.6	0.98		1.00		1.00		0.99	
N1-82/1-S	0.6	0.98		0.03		0.02		0.66	
N7-N6-N5-N4	0.9	1.00		0.45		1.00		0.98	
N6-N5-N4-N3	0.9	0.98		0.62		0.90		0.98	
N5-N4-N3-N2	0.9	0.99	0.99	0.93	0.75	0.00	0.68	0.98	0.76
N4-N3-N2-N1	0.9	1.00		0.98		0.89		0.80	
N3-N2-N1-82/1	0.9	0.98		0.98		1.00		0.44	
N2-N1-82/1-S	0.9	0.99		0.52		0.30		0.38	
N7-N6-N5-N4-N3	1.2	0.99		0.03		0.95		0.98	
N6-N5-N4-N3-N2	1.2	0.99		0.76		0.30		0.98	
N5-N4-N3-N2-N1	1.2	1.00	0.99	0.96	0.70	0.33	0.63	0.88	0.84
N4-N3-N2-N1-82/1	1.2	0.99		0.99		0.94		0.66	
N3-N2-N1-82/1-S	1.2	0.99		0.76		0.64		0.69	
N7-N6-N5-N4-N3-N2	1.5	0.99		0.20		0.58		0.99	
N6-N5-N4-N3-N2-N1	1.5	0.99	0.99	0.85	0.72	0.00	0.48	0.93	0.85
N5-N4-N3-N2-N1-82/1	1.5	0.99		0.98		0.61		0.78	
N4-N3-N2-N1-82/1-S	1.5	0.99		0.86		0.74		0.71	
N7-N6-N5-N4-N3-N2-N1	1.8	0.99		0.48		0.15		0.96	
N6-N5-N4-N3-N2-N1-82/1	1.8	0.99	0.99	0.91	0.76	0.16	0.29	0.86	0.87
N5-N4-N3-N2-N1-82/1-S	1.8	0.99		0.91		0.56		0.78	
N7-N6-N5-N4-N3-N2-N1-82/1	2.1	0.99	0.99	0.65	0.77	0.00	0.09	0.91	0.88
N6-N5-N4-N3-N2-N1-82/1-S	2.1	0.99		0.89		0.19		0.85	
N7-N6-N5-N4-N3-N2-N1-82/1-S	2.4	1.00	1.00	0.70	0.70	0.00	0.00	0.89	0.89

Changing of the horizontal and curvature gradients in N–S and E–W direction can be seen on Figs. 7 and 8 respectively. The changing of the horizontal gradients W_{zx} and W_{zy} more or less can be regarded as a linear within 30 cm distance, – as it can be seen on Figs. 7 and 8, although there are exceptions of W_{zy} on the surroundings of point 82/1 and at the N direction of 1.5 m away from this point. All these are well confirmed by the data of Tab. 7. At the same time the changing of the curvature gradients W_{Δ} and W_{xy} within a distance of 30 cm still is not considered to be linear, furthermore these are the two quantities required for the interpolation of the deflection of the vertical and for the determination of the fine structure of the geoid. Especially the changing of W_{Δ} at the surroundings of point 82/1 calls the attention to the fact, that is not to be trusted in the linearity of the curvature data still within a few dm distances, if the order of magnitude of gravity gradients is so huge as in the Mátyás cave.

5 Summary

Linear changing of the gravity gradients between the adjoining network points is an important demand for different interpolation methods in geodesy. To study the linearity of gravity gradients, torsion balance measurements were made both at the characteristic region of a Hungarian plate area at the south part of the Csepel island, and at the gravity microbase network in the gravity laboratory of Loránd Eötvös Geophysical Institute in the Mátyás cave.

The results of our investigations show that the linearity of the gravity gradients mainly depends on the point density of the torsion balance stations. It seems that the given point density of the earlier torsion balance stations generally may be not enough for some geodetic purposes. Moreover the problem could not be solved applying topographic reduction, because the mass density of the subsurface soil is extremely diverse.

Further investigations are planned to study the effects of the nonlinearity on geodetic quantities, regarding e.g. the deflection of the vertical and precise geoid computation. Investigations would be important to study the connection between the spatial structure of the gradients of gravity field and the geological fine structure of rocks near-surface inhomogeneities and shallow subsurface density.

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