HUNGARIAN CONTRIBUTION TO THE RESEARCH ON NUMERICAL SOLUTIONS AND ITS THEORIES IN MATHEMATICAL GEODESY (2015-2018) – IAG INTER-COMMISSION COMMITTEE

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Currently available high-resolution Digital Elevation Models (DEM) permit computations of terrain-related gravitational parameters with an unprecedented accuracy. It is ±0.1 mGal (10^{-6} m/s^2) and ±10 E unit (10^{-8} s^{-2}) in terms of the first and second derivatives of the gravitational potential, respectively. These grid models, however, mean a huge number of elementary polyhedron volume elements (even ~100 million polyhedrons in case of a country as small as Hungary), the computation of the gravitational effect of which is a real challenge when fully analytical solution is preferred. Although the more detailed the better principle is generally accepted it is basically true only for errorless data. As soon as errors are present any forward gravitational calculation from the model is only a possible realization of the true force field on the significance level determined by the errors. As a consequence, the computational time can be significantly reduced by the optimization of the number of volume elements based on the accuracy estimates of the input data. Common gravity field modelling programs generate optimized models for every computation point (*dynamic approach*), whereas the *static approach* provides only one optimized model for all computational points. Based on the static approach, two different algorithms were developed which may reduce the number of volume elements efficiently by a factor of 5-10 if the known/estimated accuracy of the terrain data is interpreted as a threshold parameter (Benedek et al. 2018). The grid-based algorithm starts with the maximum resolution polyhedral model defined on a uniform grid and generates a new polyhedral surface. The other algorithm is more general, it works also for irregularly distributed data (points) connected by triangulation. Beyond the description of the optimization schemes Benedek et al. (2018) presents some applications of these algorithms in regional and local gravity field modelling too.

Monte Carlo simulation and geometrical analyses were developed to investigate triangular trihedral and rounded corner reflectors, which have areas without triple-bounced backscattering (Kalmár and Bányai 2017). Removing these blind areas, the surface and the material cost (and weight) can be significantly reduced without the loss of radar cross section (RCS).

A new robust LiDAR processing method was developed (Nagy et al. 2017). This method fits a regression plane to a point cloud in any horizontal position by fitting a disc on it, which contains a specified portion of points under the disc plane in all three sectors of the disc. This method can be used to create DEMs even without any filtering process. This article also describes an analysis, which compares the results of the fitting disc method using different parameters in processing of DEMs.

The geodynamic effects of extended and heavy rainfalls on the tilt observation in Conrad Observatory, Austria (alpine terrain), was investigated by Kalmár and Benedek (2017). A grid-based finite element model was developed to determine soil water loading, water catchment area and water separation lines. Considering proper terrain conditions, realistic loadings were estimated in this test area.

The application of moving statistical functions (mean, standard deviation, skewness, peak, convolution and regression) were investigated in the paper by Kalmár (2015) for automatic detections of different events in very long time series (several hundred thousand). Methods were developed for

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detection of Q outbreaks in geomagnetic registration and for shift detections in geodynamic tilt registrations.

In the study of Bányai et al. (2015) a new generalized free network adjustment of GNSS baselines components has been implemented together with simultaneous estimation of seven parameters of similarity transformation relative to the coordinates of properly chosen stations estimated in the reference epoch. The similarity parameters are handled by constraint equations and the more stable singular value decomposition is used for the inversion of the extended normal equations. This procedure is proposed for estimation of relative displacements in local or regional networks if they are near to plate borders and only far GNSS fiducial stations are available.

The displacements in the line of sight directions between the SAR satellites and the moving persistent scatters, estimated by satellite radar interferometry, can be handled as geometric observables. Bányai et al. (2017) investigate the geometric features of ascending and descending changes - along with possible fusion of other geodetic observables, which can be used to estimate three-dimensional displacements. The accuracies, correlations and dilution of precisions of the fusion are estimated and the combination with GNSS displacements using the Kalman-filter is proposed.

Spherical harmonics are the most widely spread mathematical tool for describing the gravity field, generating a frequent need for spherical harmonic synthesis and spherical harmonic analysis. The huge amount of computational load can be efficiently reduced by certain numerically optimal solutions, which were efficiently implemented in a C++ script (Kemény and Földváry 2015).

Legendre polynomials of the spherical harmonics are defined in classical geodetic applications with the Rodriguez formula. It has been shown analytically that Legendre polynomials can also be considered as a finite set of periodic signals (Földváry 2015), which can efficiently simplify investigations of periodic component of the signal.

It is known that the averaging method makes smoother the extremes. Accordingly, estimation of the periodicity of an averaged time series is influenced by the smoothing property of the averaging, resulting in an underestimated amplitude of the periodicity. This can be partially recovered by an appropriate de-smoothing method (Földváry 2015). The same de-smoothing method can be interpreted for revoking of spatial smoothing effect of raster data, c.f. Digital Terrain Models (Földváry 2018).

The advantages of GIS software in handling, interpretation and analysis of geospatial data, have not been adopted for the satellite gravimetry before. First attempts for implementing satellite gravimetric data in GIS are provided by Földváry et al (2015). Also, the advantages of GIS has been implemented for describing the seismology of the Moon. As a result, an online GIS service has been developed under the name HGR.01, which contains all relevant information of the Moonquake observations so far (Lázár et al 2018a, b).

In the area of mathematical Geosciences, a new book was written and published about the hybrid symbolic-numeric methods in Mathematical Geosciences (Awange et al. 2018). Hybrid symbolic-numeric computation is a large and growing area at the boundary of mathematics and computer science. Three major areas of computation are covered in Awange et al, 2018. The first part discusses methods for computing all solutions to a system of polynomials. Purely symbolic methods e.g. via Gröbner bases tend to suffer from algorithmic inefficiencies, and purely numeric methods such as Newton iterations have trouble finding all solutions to such systems. One class of hybrid methods blends numerical calculations into the purely algebraic approach e.g. computing numeric Gröbner bases or Dixon resultants (the latter being extremely efficient e.g. for elimination of variables). Another hybrid technique mixes symbolic methods into more numerical approaches, e.g. finding initializations for numeric homotopy tracking to obtain all solutions. The second part goes into the realm of "soft" optimization methods, including genetic methods, simulated annealing, and particle swarm optimization, amongst others. These are all popular and heavily used, especially in the context of global optimization. While often considered as "numeric" methods, they benefit from symbolic computation in several ways. One is that implementation is typically straightforward when one has access to a language that supports symbolic computation. Updates of state, e.g. to handle mutations and gene crossover, are easily coded. Indeed, this sort of thing can be so deceptively simple baked into the language so to speak, that one hardly realizes symbolic computation is happening. Amongst many applications in this part there is, again, that of solving systems of equations. Also, the mixed-integer programming is covered wherein some variables are discrete-valued and others continuous. This is a natural area for Hybrid symbolic-numeric computation since it combines aspects of exact and numeric methods in the handling of both discrete and continuous variables. This technique can be employed to solve GNSS phase ambiguity problem. The third part delves into data modelling. This begins with use of radial basis functions and proceeds to machine learning, e.g. via Support Vector machine methods. Symbolic Regression, a methodology that combines numerical calculations with evolutionary programming, is also introduced for the purpose of modelling data. Another area seeing recent interest is that of robust optimization and regression, wherein one seeks results that remain relatively stable with respect to perturbations in input or random parameters used in the optimization. Several hybrid methods are presented to address problems in this realm. Stochastic modelling is also discussed. This is yet an-other area in which hybrid methods are quite useful.

Symbolic computing languages have seen a recent trend toward ever more high-level support for various mathematical abstractions. This appears for example in exact symbolic programming involving probability, geometry, tensors, engineering simulation, and many other areas. Under the hood is a considerable amount of Hybrid symbolic-numeric computation. Naturally such support makes it all the easier for one to extend hybrid methods; just consider how much less must be built from scratch to support, say, stochastic equation solving, when the language already supports symbolic probability and statistics computations. Awange et al (2018) present to the reader some of the major areas and methods that are being changed, by the authors and others, in furthering this interplay of symbolic and numeric computation. The term Hybrid Symbolic-Numeric Computation has been with us for over two decades now.

A brief list of topics covered:

- Systems of polynomial equations with resultants and Gröbner bases
- Simulated annealing
- Genetic algorithms
- Particle swarm optimization
- Integer programming
- Approximation with radial basis functions
- Support vector machines
- Symbolic regression
- Quantile regression
- Robust regression
- Stochastic modelling
- Parallel computations

Most of the methods discussed in book will probably be implemented by the reader on a computer algebra system. The two most fully developed and widely used computer algebra system are Mathematica and Maple. Some of the polynomial computations here are better done on the specialized system Fermat. Other systems worthy of mention are Singular and SageMath.

The book by Awange et al. (2018) is not a reference manual for any system, and we have made an effort to keep the specialized commands to a minimum, and to use commands whose syntax makes them as self-explanatory as possible. More complete Mathematica programs to implement some of the examples are available online. Similarly, a program written in Fermat for the resultant method called Dixon-EDF is available online.

A novel RANSAC robust estimation technique has been investigated as an efficient method for solving the 7-parameter datum transformation problem in the presence of outliers (Paláncz et al. 2017). A new dual quaternion representation of the 3-point similarity transformation problem and its non-iterative fast solution via linear homotopy has been developed. Paláncz at al. (2017) show how to build this model into a RANSAC shell and how to solve it in parallel way decreasing the computation time further. In addition, an efficient strategy based on the early stopping principle was given for adjusting automatically the threshold level of the robust algorithm properly. This method, which is frequently employed in geodesy has two sensitive features (i) the user adjusts some parameters of the algorithm making it subjective and a rather difficult procedure and (ii), its shell should repeatedly solve a nonlinear equation system. In this contribution the authors suggested an automatic adjustment strategy for the most important parameter ``the threshold value'' based on the "early stopping" principle of the machine learning technology. Using iterative numerical methods, they proposed the use of an algebraic polynomial system developed via dual quaternion technique and solved by non-iterative homotopy method thereby reducing the computation time considerably.

In order to illustrate the proposed method, the transformation parameters of the Western Australian Geodetic Datum (AGD 84) to Geocentric Datum Australia (GDA 94) was computed (Paláncz et al. 2017). The suggested algorithm requires longer running time than the standard Procrustes algorithm, however our optimal RANSAC is a robust method eliminating outliers. The novelty of the proposed approach lies in three major contributions (i) the provision for automatically finding the proper error limit parameter for RANSAC method, which has until now been an error-trial technique, (ii) employing algebraic polynomial form of the dual quaternion solution in the RANSAC shell thereby accelerating the repeatedly requested solution process and (iii) avoiding iterations via a heuristic approach of the scaling parameter.

Paláncz and Awange (2017) introduce the nonlinear homotopy in geodesy. Never had the concept of nonlinear homotopy been used by the geodetic community. This is partly attributed to the complexity of the involved equations and partly due to the computational time required. Recently, however, Nor et al. (2013) suggested the possibility of constructing nonlinear homotopy. The idea of vhich is developed for geodetic applications and an example of its use illustrated. Awange and Paláncz (2016) is dealing with the problem that the improved geo-spatial instrumentation and technology such as in laser scanning has now resulted in millions of data being collected, e.g., point clouds. It is in realization that such huge amount of data requires efficient and robust mathematical solutions that this third edition of the book extends the second edition by introducing three new chapters: Robust parameter estimation, Multiobjective optimization and Symbolic regression. Furthermore, the linear homotopy chapter is expanded to include nonlinear homotopy. These disciplines are discussed first in the theoretical part of the book before illustrating their geospatial applications in the applications chapters where numerous numerical examples are presented. The renewed electronic supplement contains these new theoretical and practical topics, with the corresponding Mathematica statements and functions supporting their computations introduced and applied. This third edition is renamed in light of these technological advancements.

In indoor and outdoor navigation, finding the local position of a sphere in mapping space employing a laser scanning technique with low-cost sensors is a very challenging and daunting task Lewis et al. (2018a) illustrate how Gröbner basis techniques can be used to solve polynomial equations arising when algebraic and geometric measures for the error are used. The effectiveness of the suggested method is demonstrated, thanks to standard CAS software like Mathematica, using numerical examples of the real world.

Improvements in computational and observational technologies in geoinformatics, e.g., the use of laser scanners that produce huge point cloud data sets, or the proliferation of GNSS and unmanned aircraft vehicles (UAVs), have brought with them the challenges of handling and processing this “big data”. These call for improvement or development of better processing algorithms. One way to do that is integration of symbolically presolved subalgorithms to speed up computations. Using examples of interest from real problems in geoinformatics Lewis et al. (2018b) discuss the Dixon-EDF resultant as an improved resultant method for the symbolic solution of parametric polynomial systems. The method itself is briefly described, then geoinformatics problems arising in minimum distance mapping (MDM), parameter transformations, and pose estimation essential for resection are discussed. Dixon-EDF is then compared to older notions of "Dixon resultant", and to several respected implementations of Gröbner bases algorithms on several systems. The improved algorithm, Dixon-EDF, is found to be greatly superior, usually by orders of magnitude, in both CPU usage and RAM usage. It can solve geoinformatics problems on which the other methods fail, making symbolic solution of parametric systems feasible for many problems.

In the mathematical geodesy the different approaches of the tree-dimensional coordinate transformations are essential topics since they are frequently applied in the geodetic and surveying practice. Besides the traditional iterative solutions there are new closed form solutions, which consider the advantages of new mathematical algebra concept. Závoti and Kalmár (2016) present different solutions of the three-dimensional seven parameter Bursa-Wolf model which are based on quaternions and skew symmetric rotation matrices.

Quaternion-based geodetic datum transformation by iteration was developed (Papp 2015). Datum transformation is the most frequent problem in geodesy, photogrammetry, geoinformatics, and animation and computer vision. To overcome the drawback of traditional solution of the problem based on rotation angles this implementation adopts unit quaternion to represent three-dimensional rotation matrix. A quaternion-based iterative solution in terms of linearization in the Bursa-Wolf geodetic transformation model is described. The calculations show that the quaternion-based solution has no dependence on the initial values of the parameters. It provides reliable result with fast speed. The main advantage of this algorithm is that it can be applied in the case of arbitrary size rotation.

In Bursa-Wolf seven parameter similarity transformation model the transformation consists of three translation parameters, three rotation elements and one scale factor. Papp (2017) proposes a closed-form solution of datum transformation. In this solution, Clifford dual-number quaternions are used to represent the 3D rotation. Once the transformation parameters between the two data are established by identical point pairs, the rotation matrix, the scale parameter and the translation vector are simultaneously derived. The proposed algorithm seems better than the iterative algorithms. Most importantly, in contrast to unit quaternion-based algorithms, the presented algorithm solves seven unknown parameters simultaneously without the initial estimates of unknowns. The main advantage of this algorithm is that it can be applied in case of arbitrary size rotations. Consequently, the mathematical modelling of similarity transformation based on dual quaternions is an elegant method which is adaptable to present a compact formula for Bursa-Wolf model (Papp 2017).

An application aiming at coordinate transformation of LiDAR point clouds between the Hungarian Datum 1972 and the ETRS89 has been developed by Brolly (2018). The first step is a 7-parameter Helmert transformation with country-wide parameter set, which is refined by using a correction grid from accurate local transformations. The application allows direct read and write for ASPRS standard LAS file format and delivers transformation accuracy of a few centimetres throughout Hungary.

A new method for error detection with scale factors based on coordinates of model and control points was developed (Jancsó 2017). In photogrammetry the model coordinates of control points can be calculated, and they can be used effectively to detect gross errors located in control points. To reach this goal triangles are formed from points in every combination and the residuals of scale factors are calculated. The triangle where the sum of squared scale factor residuals is the smallest one can be found. Adding other points to this triangle one by one tetrahedrons are formed. Using the edges of these tetrahedrons equations of constraint conditions are compiled and the geodetic coordinates of the examined point can be determined. These calculated coordinates can be compared to the given geodetic coordinates and the allowable residuals are calculated. By this way all points can be filtered unanimously against gross errors.

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