GEOID COMPUTATIONS BASED ON TORSION BALANCE MEASUREMENTS

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Abstract

A very large part of the area of Hungary - first of all flatland and hilly areas of moderate height - was covered with a network of torsion balance stations. As far as torsion balance measurements are considered Hungary is the most well measured country in the world. This gives us a good possibility to interpolate a very dense net of deflections of the vertical from gravity gradients and applying astronomical levelling to compute geoid heights. Results of the computations in a Hungarian test area show mean square errors below a decimetre of geoid undulations computed in this way.

Introduction

Nowadays based on up-to-date geodetic measurement results, contour line maps of the main geoid forms for the whole Earth are available. These global geoid forms - having a few metres accuracy - however, do not contain the "fine structure" of the geoid, while we need at least centimetres accuracy for the practical geodetic purposes (e.g. determination of height above see level from GPS measurement). A dense net of deflections of the vertical interpolated from gravity gradients measured by torsion balance gives us a good possibility to determine a detailed local geoid map.

As known, there exists a definite mathematical relationship between the geoid undulations and the deflections of the vertical (*BIRÓ, 1982*). Between any points P_i and P_k the geoid height change is

$$\Delta N_{ik} = \int_i^k (\xi \cos\alpha + \eta \sin\alpha) ds \qquad (1)$$

where α is the azimuth of the line of length s connecting the two points. For using this line integral we would have to know the function of deflections of the vertical between the points. In practice the integral (1) is to be evaluated by a numerical integration:

$$\Delta N_{ik} = \left(\frac{\xi_i + \xi_k}{2} \cos\alpha_{ik} + \frac{\eta_i + \eta_k}{2} \sin\alpha_{ik} \right) s_{ik} \quad . \tag{2}$$

Generally we want to compute the difference ΔN not only between two points but we need a detailed geoid map for a larger territory. If we want to apply astronomical levelling for a larger territory, the deflection components must be given at enough stations such that the interpolation between these stations can be done reliably. In practice we have a sparser net of astronomical stations, and this astrogeodetic net is interpolated by different methods. There is a very good possibility to interpolate deflections of the vertical from gravity gradients where torsion balance measurements are available.

A very simple relationship based on potential theory can be written for the changes of $\Delta\xi_{ik}$ and $\Delta\eta_{ik}$ between arbitrary points i and k of the deflection of the vertical components ξ and η as well as for gravity gradients W_Δ and W_{xy} measured by torsion balance:

$$\begin{aligned} &\Delta\xi_{ik} \sin\alpha_{ki} - \Delta\eta_{ik} \cos\alpha_{ki} = \\ &\frac{n_{ik}}{4g} \left\{ \left[(W_\Delta - U_\Delta)_i + (W_\Delta - U_\Delta)_k \right] \sin 2\alpha_{ki} + \left[(W_{xy} - U_{xy})_i + (W_{xy} - U_{xy})_k \right] 2\cos 2\alpha_{ki} \right\} \end{aligned} \tag{3}$$

where n_{ik} is the distance between points i and k, g is the average value of gravity between them, U_Δ and U_{xy} are gravity gradients in the normal gravity field, whereas α_{ki} is the direction azimuth between the two points (*VÖLGYESI, 1993, 1995*). The computation being fundamentally an integration, practically possible only by approximation, in deriving (3) it had to be supposed that the change of gravity gradients between points i and k, measurable by torsion balance, was linear − thus the equality sign in (3) is valid only for this case (*VÖLGYESI, 1993*).

If we have a denser net of deflections of the vertical, and we want to apply astronomical levelling for a larger territory, we should first interpolate deflection of the vertical components ξ and η from an arbitrary shaped network points to grid points of a square-shaped network. Then applying equation (2) we can compute differences of geoid heights, along sets of points of the square-shaped network in both east-west and north-south profiles. Going around each square of the square-shaped network and summarising ΔN differences we can get misclosures. These misclosures can be adjusted as if they were the misclosures of the well known geometrical levelling. Results of this computation will be the adjusted ΔN differences between the adjacent square-shaped grid points. If an initial geoid height N_0 is given at an arbitrary point of the grid we can get the final geoid heights N_i at point i, summarising the suitable ΔN differences.

A computer program package developed by us is able to determine deflections of the vertical based on torsion balance measurements either along triangulation chains or in networks covering arbitrary areas using any of the interpolation methods fully described in (VÖLGYESI, 1993). It can plot the interpolation network and vector diagram of interpolated deflections of the vertical, calculate geoid heights by astronomic levelling and also plot either a perspective or an isoline map of the geoid for the area.

Test computations

A characteristic Hungarian area measured by torsion balance extended over some 1200 km^2 was chosen for the purpose of our test computation. The torsion balance measurement points' location in our test area is displayed in *Fig. 1.* Stations were not located with the same density because the observations were carried out with a greater density of points in "disturbed" areas of rugged topography. In *Figs. 2* and *3* gravity gradients W_Δ and W_{xy} measured by torsion balance were plotted on isoline maps.

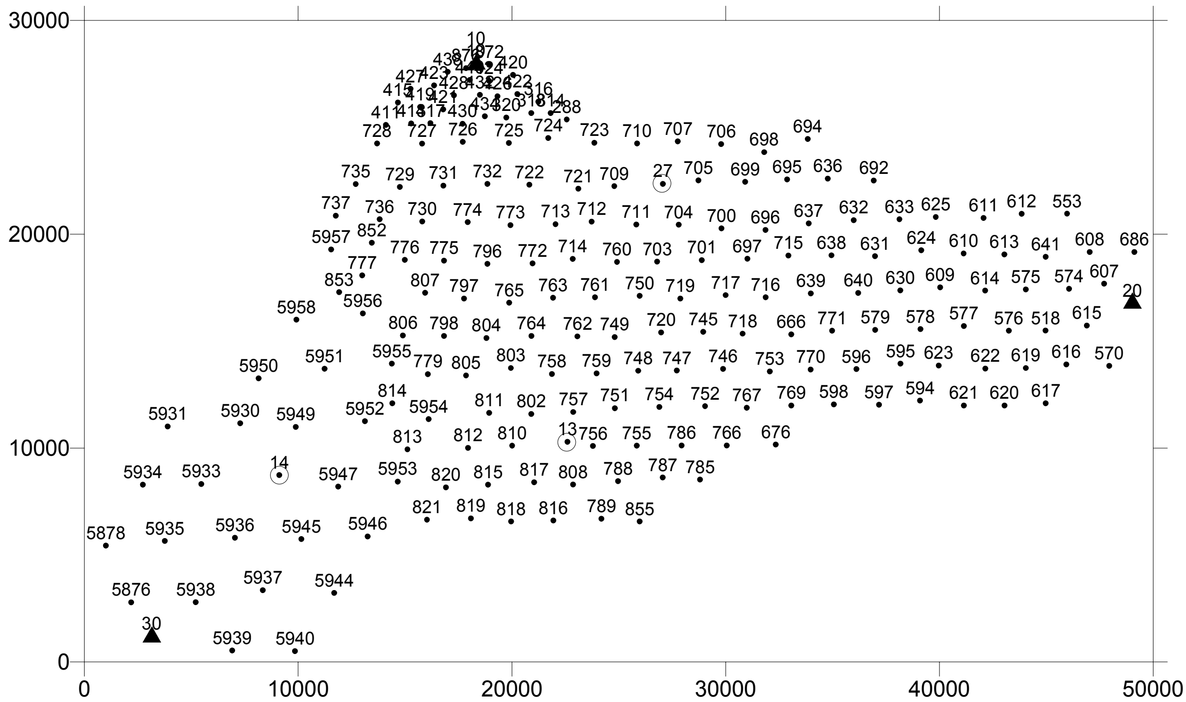

Fig. 1. *Torsion balance stations*

Six points can be found in the test area where x , h deflections of the vertical are known. Each of these points is such that gravimetric deflections of the vertical are available based on gravity data; four of them (points labelled *10*, *20*, *30*, and *27*) are astrogeodetic points. Points *10*, *20* and *30* are fixed points of interpolation, points *13*, *14* and *27* served the purpose of checking interpolated values. The accuracy of relative deflections of the vertical at the astrogeodetic stations *10, 20, 30* and *27* can be described by the standard error of astronomic position determinations, which is $\mu_{\xi o} \approx \mu_{\eta o} \approx \pm 0.2''$.

The interpolation network in *Fig. 1* has 206 points in all and 203 of these are points with unknown deflections. Since there are two unknown components of deflection of the vertical at each point there are 406 unknowns for which 558 equations can be written.

In *Figs. 4* and 5 *x* and *h* components of deflections of the vertical are visualized in isoline maps that resulted from the computation. Besides this it is given in *Table 1* how large deviations arose at checkpoints between *computed* and *known* *x , h* values. Standard deviations $m_\xi = \pm 0.60''$ and $m_\eta = \pm 0.65''$, computed from these departures at checkpoints corroborate the fact that even for large continuous *x , h* values of acceptable accuracy can be computed from torsion balance measurements.

If for any point of the investigated area the initial value of geoid height N_0 is known, further if adequately interpolated deflection values are available for this particular area, the detailed map of the geoid can be obtained for the given area. Based on the above, computations were carried out for our experimental area shown in *Fig. 1*, using the deflection components interpolated previously.

Table 1

Checking point	δξ"	δη"
27	-0.69	-0.51
13	+0.54	+0.96
14	+0.55	+0.29
Standard deviations	±0.60	±0.65

As on this territory no geoid height related to the same reference surface as the given deflection components was available, the geoid height N_0 for the experimental area in *Fig. 1* was chosen arbitrarily to be *40.00 m* in the astrogeodetic point *30*, to serve as basis for determining geoid heights of further points.

The mentioned geoid map is presented in *Fig. 6.* It is characteristic for the accuracy of the computations that going along the chains *30 → 10, 10 → 20, 20 → 30,* and returning to point *30*, instead of the initial value N_{30} *= 40.00 m*, N_{30} *= 39.93 m* was obtained, - i.e. going around the chain length of about 115 km, a misclosure of only 7 cm was obtained. This misclosure is characteristic not only of the accuracy of the geoid height, but is at the same time an excellent possibility to check the confidence of the interpolated ξ and η values. Therefore deflection values interpolated from torsion balance observations can be stated to give very economically highly reliable geoid maps which are most suitable to study local details.

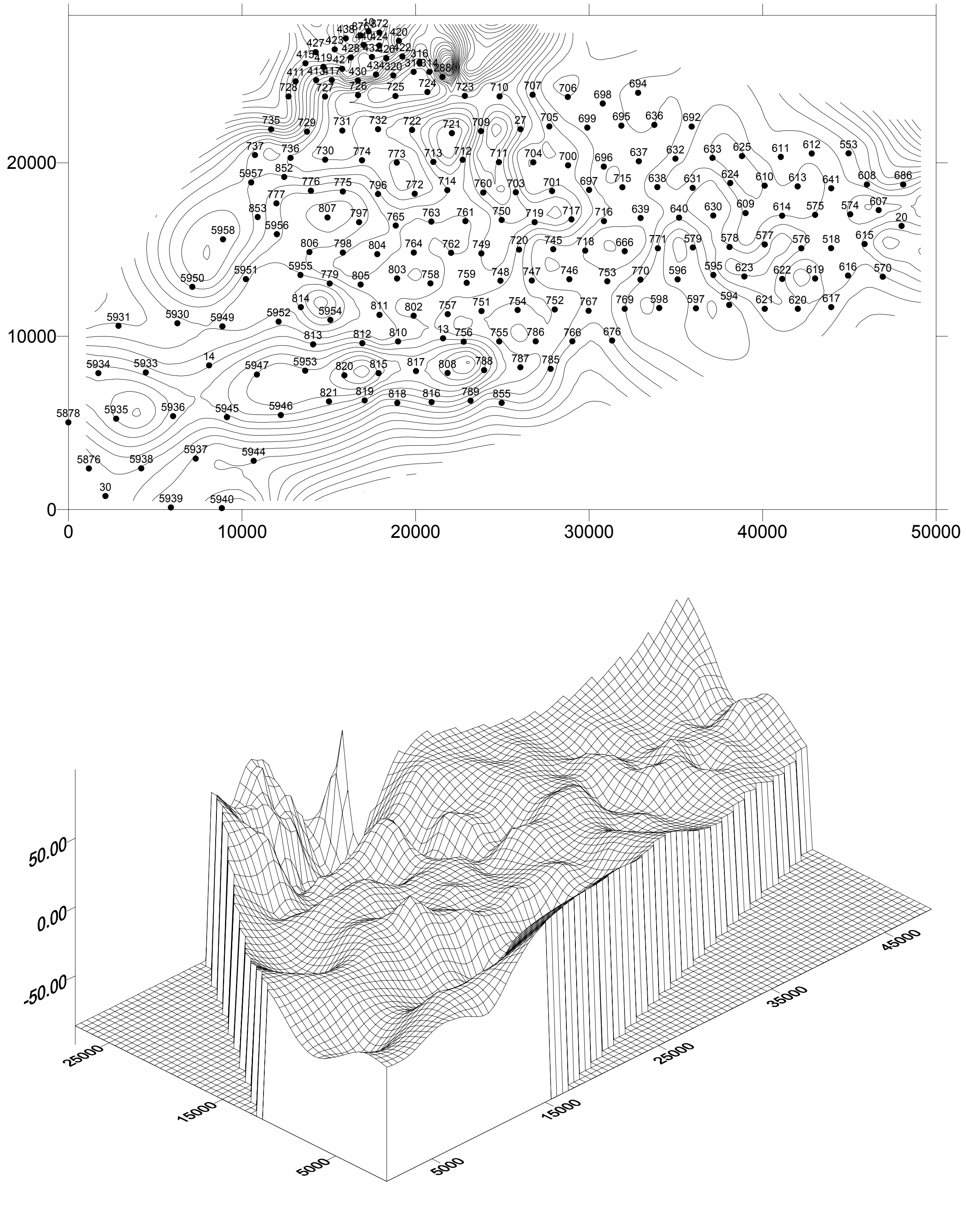

Fig. 2. W_{Δ}

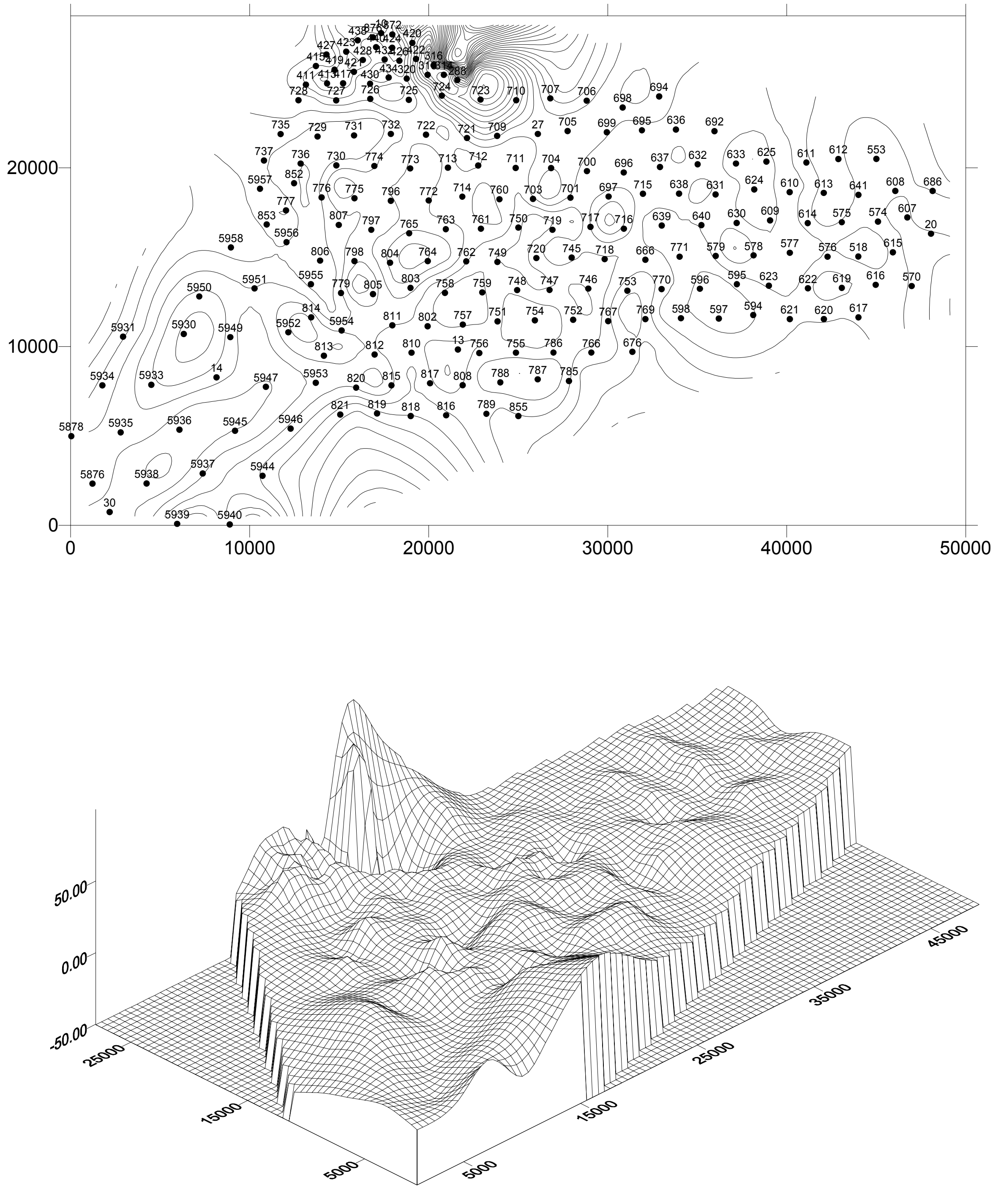
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Fig. 3. W_{xy}

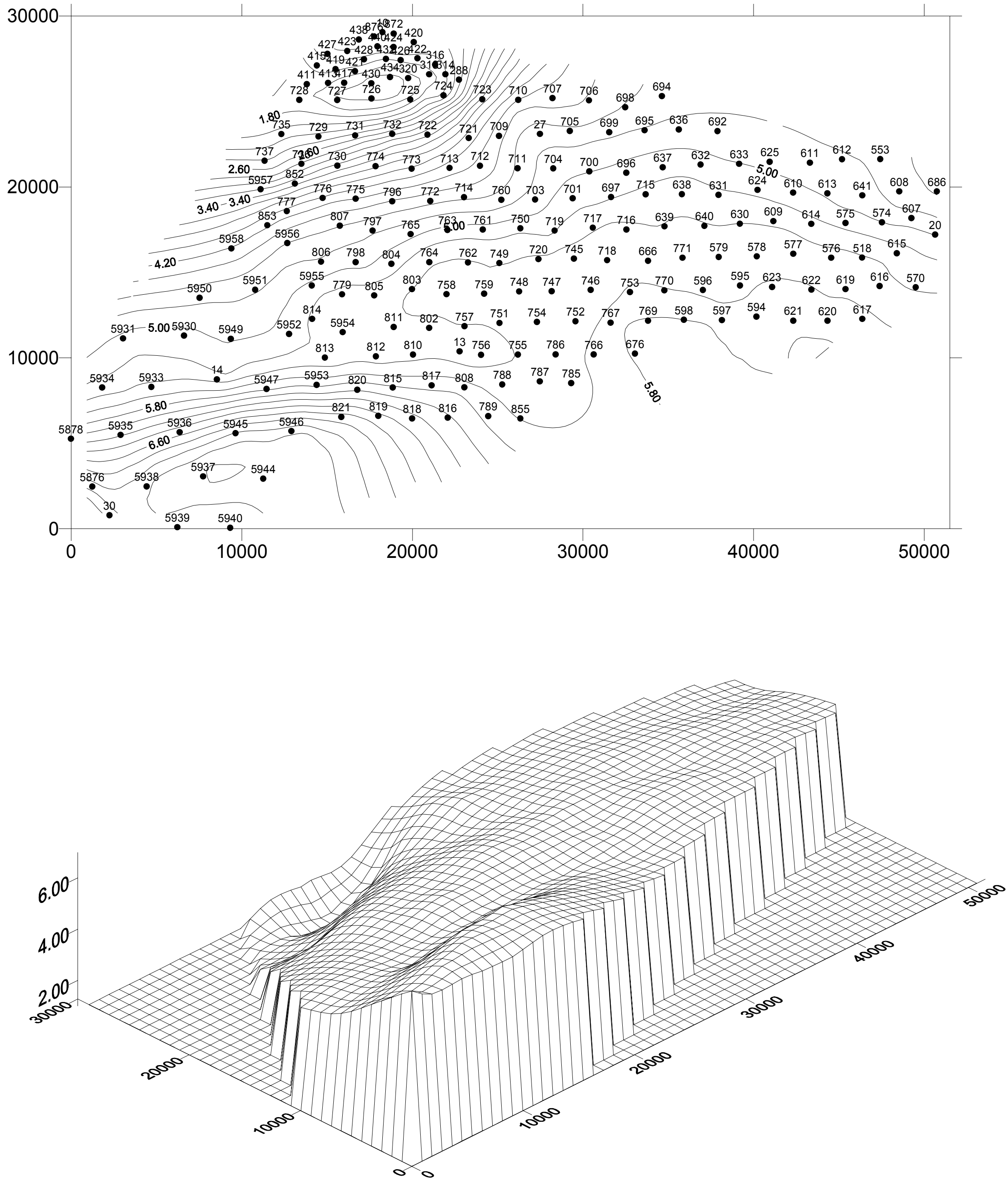

Fig. 4. *Deflection of the vertical (ξ component)*

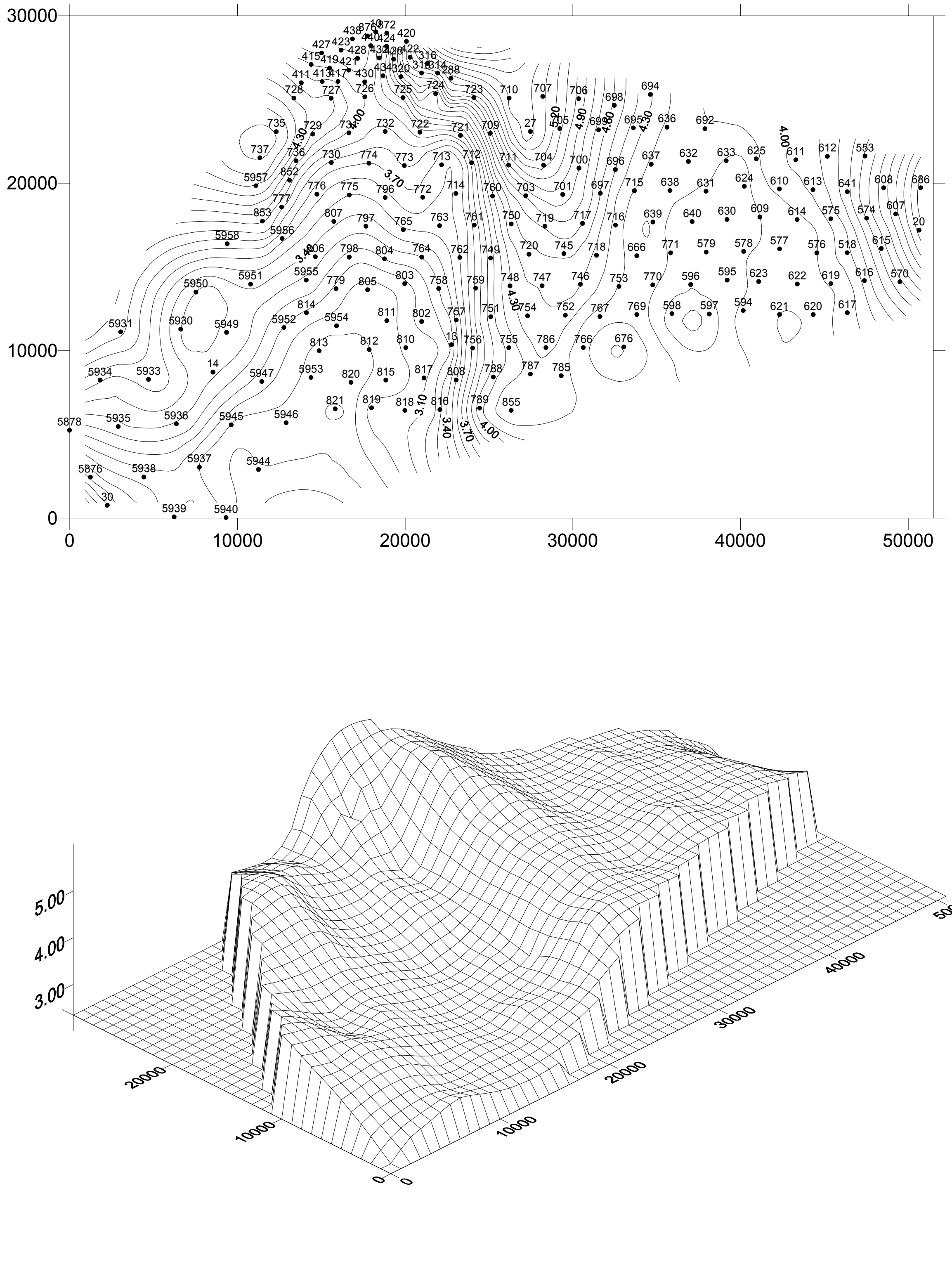

Fig. 4. *Deflection of the vertical (η component)*

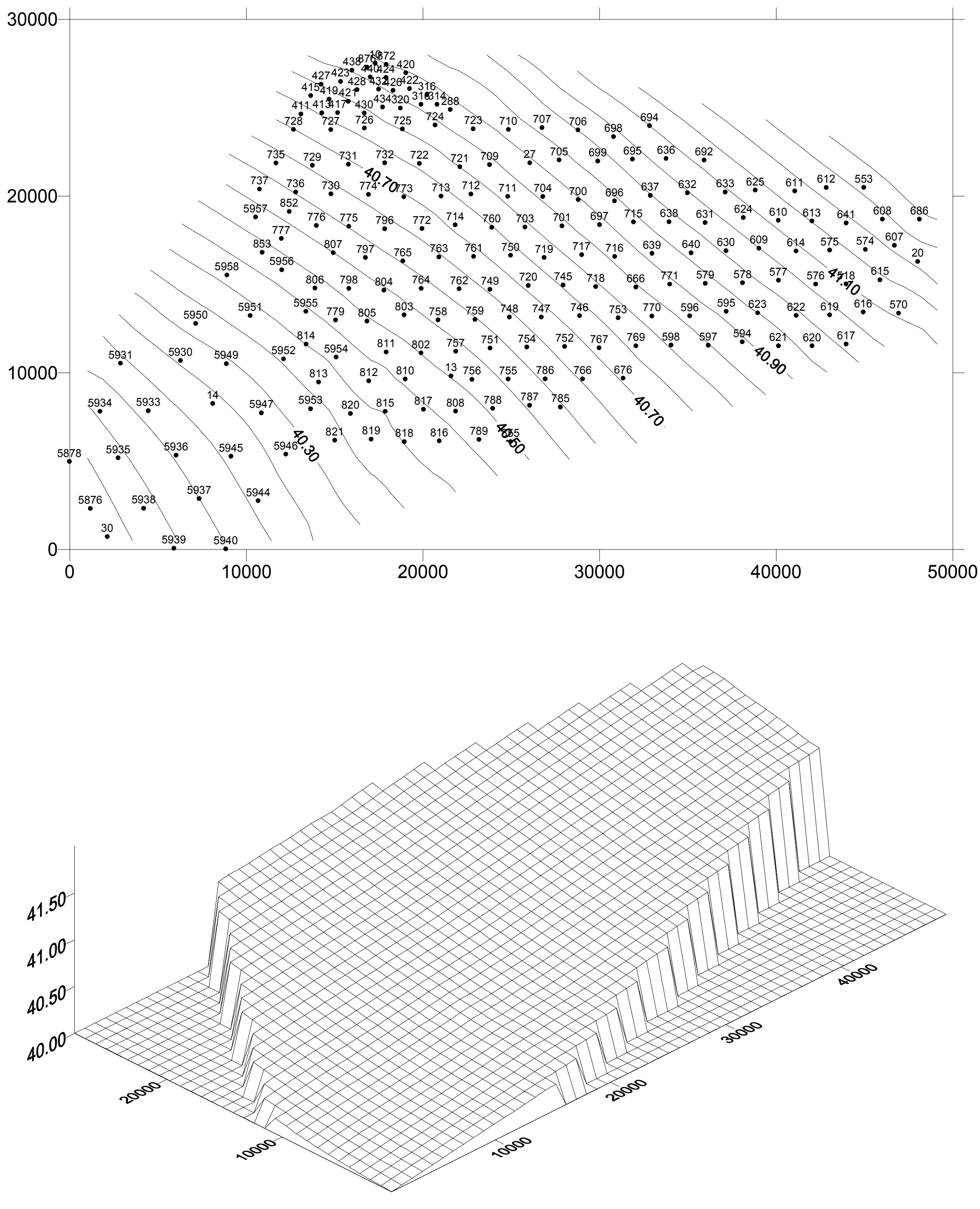

Fig. 4. *Geoid heights*

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