Local gravity field modeling using surface gravity gradient measurements

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Abstract. Almost 100,000 surface gravity gradient measurements exist in Hungary over an area of about 45 000 km^2. These measurements are a very useful source to study the short wavelength features of the local gravity field, especially below 30 km wavelength. Our aim is to use these existing gravity gradient data in gravity field modeling together with gravity anomalies. Therefore we predicted gravity anomalies from horizontal gravity gradients using the method of least-squares collocation. The cross-covariance function of gravity gradients and gravity anomalies was estimated over the area and a suitable covariance model was estimated for the prediction. The full covariance matrix would require about 15 GB storage, however, the storage requirement can be reduced to about 300 MB by inspecting the structure of the cross-covariance function. Using sparse linear solvers the computation proved to be manageable, and the prediction of gravity anomalies for the whole area was performed. The results were evaluated at those sites where Δg values were known from measurements in the computational area.

Keywords: horizontal gradients of gravity, gravity gradient measurements, least-squares collocation, sparse matrix solvers

1 Introduction

In view of the increasing accuracy demands of local gravity field determination in the GPS era, it seems advantageous to combine all available measurements to the gravity field for the purpose. Since our knowledge of the local gravity field is based mainly on gravity measurements, the combination of other kind of gravity field parameters (e.g. observations on the direction of the gravity vector or its horizontal gradient) with gravity measurements is preferable.

Several authors developed methodologies to combine horizontal gravity gradient and gravity measurements for local gravity field determination. Vassiliou, for example, showed how to process and downward continue airborne gradiometer data (Vassiliou, 1986). Hein discussed many ways of dealing with gravity gradient measurements that are available in Germany in the Upper Rhine Valley (Hein, 1981). He processed altogether 21616 such measurements in view of local gravity field determination. Another method, the so-called gradient kriging with terrestrial gravity gradients was proposed and used by Menz and Knospe (2002) for local gravity field determination.

The problem is particularly interesting to us since in Hungary we have almost 100,000 surface gravity gradient measurements. Our aim is therefore to combine these measurements with gravity data as well as other data in view of a new geoid solution (Völgyesi et al, 2004). This is the reason why the least-squares collocation method was chosen, since it is well known that within this method it is relatively easy to process different kind of gravity field parameters in a theoretically sound framework. The main problem of least-squares collocation is that it is computationally demanding as it requires the solution of a linear system of which the number of unknowns equal to the number of measurements. Therefore several authors proposed compactly supported covariance functions that lead to sparse matrix techniques to reduce the computational burden (Sansò and Schuh, 1987).

First, we briefly review the necessary details of the least-squares collocation method. Next within the framework of an application example (involving an area of about 45 000 km^2) the chosen method is investigated. Finally the results are discussed and several conclusions are drawn.

2 Optimum estimation of gravity anomalies

The well-known method of least-squares collocation (Moritz, 1980) is proven to be suitable in gravity field modeling, since it allows estimating any gravity field parameter from measurements of other gravity field parameters. The prediction s at any point is obtained through the following linear system

$$s = C_{s\ell}(C_{ss} + C_{nn})^{-1}\ell\,, \qquad (1)$$

where ℓ is the measurement vector, C_{ss} and C_{nn} denote signal and noise covariance matrices, respectively, and $C_{s\ell}$ is the cross-covariance matrix of measured and predicted quantities.

In our case now we would like to predict gravity anomalies, i.e. $s = \Delta g(Q_k)$ at points Q_k, ($k = 1, \ldots K_{max}$) from measured horizontal gravity gradients V_{xz} V_{yz} at points P_i, ($i = 1, \ldots N_{max}$) i.e. $\ell = [V_{xz}(P_i)\ V_{yz}(P_i)]$. The necessary isotropic covariances can be written as functions of distance d and azimuth α (counted anticlockwise from East towards North) between any pair of points in the local x = East, y = North system as follows: $C_{Vxz,Vxz}(d, \alpha)$, $C_{Vyz,Vyz}(d, \alpha)$, $C_{Vxz,Vyz}(d, \alpha)$, $C_{\Delta g,Vxz}(d, \alpha)$ and $C_{\Delta g,Vyz}(d, \alpha)$.

The auto- and cross-covariance functions of horizontal gravity gradients $C_{Vxz,Vxz}(d, \alpha)$, $C_{Vyz,Vyz}(d, \alpha)$, $C_{Vxz,Vyz}(d, \alpha)$ are obtained by differentiation from auto-covariance function of gravity anomalies $C_{\Delta g,\Delta g}(d) = C(\Delta g, \Delta g)$

$$\begin{aligned} C_{Vxz,Vxz}(d,\alpha) &= -\frac{\partial^2}{\partial x^2}C(\Delta g,\Delta g) \\ C_{Vxz,Vyz}(d,\alpha) &= -\frac{\partial^2}{\partial x \partial y}C(\Delta g,\Delta g)\,. \\ C_{Vyz,Vyz}(d,\alpha) &= -\frac{\partial^2}{\partial y^2}C(\Delta g,\Delta g) \end{aligned} \qquad (2)$$

Also the cross-covariances $C_{\Delta g,Vxz}(d, \alpha)$ and $C_{\Delta g,Vyz}(d, \alpha)$ can be written similarly

$$\begin{aligned} C_{\Delta g,Vxz}(d,\alpha) &= \frac{\partial}{\partial x}C(\Delta g,\Delta g) \\ C_{\Delta g,Vyz}(d,\alpha) &= \frac{\partial}{\partial y}C(\Delta g,\Delta g) \end{aligned}\,. \qquad (3)$$

The linear system (1) is composed of six covariance matrix blocks $C_{xz,xz}$, $C_{yz,yz}$, $C_{xz,yz}$, $C_{yz,yz}$, $C_{xz,\Delta g}$, $C_{yz,\Delta g}$:

$$\begin{aligned} C_{\ell\ell} &= C_{ss} + C_{nn} = \begin{bmatrix} C_{xz,xz} & C_{xz,yz} \\ C_{yz,xz} & C_{yz,yz} \end{bmatrix} + C_{nn} \\ C_{s\ell} &= \begin{bmatrix} C_{xz,\Delta g} & C_{yz,\Delta g} \end{bmatrix} \end{aligned}\,, \qquad (4)$$

the blocks each contain covariance functions (2) and (3), evaluated at the distance $d(i, k)$ and azimuth $\alpha(i, k)$ of measurement and/or prediction points P_i, P_k and Q_i, Q_k, respectively. The solution of Eq (1) is an optimum estimation in the least-squares sense to gravity anomalies Δg.

An important restriction on the choice of the covariance function $C(\Delta g,\Delta g)$, besides its positive definiteness, is the existence of its six derivatives defined in Eqs. (2) and (3). A simple analytical covariance function model, which behaves well under repeated differentiation and is physically possible, is the two-parameter *Gaussian* covariance function

$$C(d) = Ae^{-Bd^2}\,. \qquad (5)$$

The covariance functions (2) and (3) can be obtained immediately as

$$\begin{aligned} -\frac{\partial C}{\partial x} &= 2ABd\,e^{-Bd^2}\cos\alpha \\ -\frac{\partial C}{\partial y} &= 2ABd\,e^{-Bd^2}\sin\alpha \\ -\frac{\partial^2 C}{\partial x^2} &= AB\,e^{-Bd^2}[1 - 2Bd^2 - (1 + 2Bd^2)\cos 2\alpha]\,. \\ -\frac{\partial^2 C}{\partial x \partial y} &= -AB\,e^{-Bd^2}(1 + 2Bd^2)\sin 2\alpha \\ -\frac{\partial^2 C}{\partial y^2} &= AB\,e^{-Bd^2}[1 - 2Bd^2 + (1 + 2Bd^2)\cos 2\alpha] \end{aligned} \qquad (6)$$

All of the above covariance functions are azimuth-dependent (non-isotropic). However, it is possible to introduce the isotropic functions

$$\begin{aligned} -\frac{\partial C}{\partial d} &= 2ABd\,e^{-Bd^2} \\ \frac{\partial^2 C}{\partial d^2} &= 2AB(1 - 2Bd^2)\,e^{-Bd^2} \end{aligned}\,, \qquad (7a,b)$$

which are useful to estimate the parameters A and B in (5) from measured horizontal gravity gradients (Tscherning, 1976). The isotropic covariance functions (7) are illustrated for the parameters A = 6.5 mGal^2 and the parameter B, implicitly defined through the correlation length $d_0 = \sqrt{\ln 2 / B}$ = 6 km in Fig. 1. The correlation length d_0 is by definition the distance where the covariance is half of the variance $C(0)$ (i.e. $C(d_0) = 0.5C(0)$).

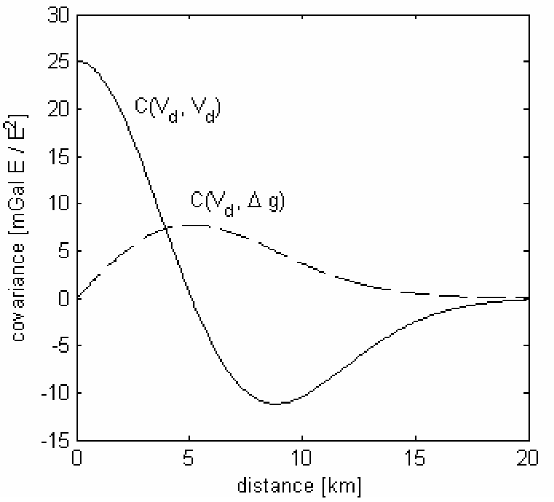

Fig. 1 Example of isotropic Gaussian cross-covariance function (7a) of gravity anomalies and horizontal gravity gradients $C(V_d, \Delta g)$ as well as isotropic auto-covariance function (7b) of horizontal gravity gradients $C(V_d, V_d)$. The parameters of the covariance function for this example are $A = 6.5$ mGal^2, $d_0 = 6$ km.

The two parameters necessary to define a Gaussian covariance function can be estimated from the empirical isotropic auto-covariance function (7b) of horizontal gravity gradients V_{xz}, V_{yz}. Since from (7b) the variance is $2AB$, and the correlation length d_g is connected to d_0 according to the formula $d_g = 0.532\ d_0$, one can take these two parameters for the estimation of A and B. The actual gravity gradient data in Hungary, which were reduced to the normal and topographic effects, show an average variance 125 E^2, whereas the correlation length d_g is about 0.7-1 km. Hence the parameters of the Gaussian covariance function (5) are approximately $A = 0.8$-3 mGal^2 and $d_0 = 1$-2 km.

3 An application example

Our example application of the collocation equation (1) is the estimation of gravity anomalies from the surface gravity gradient dataset of Hungary (Völgyesi et al., 2004). The dataset contains 44 818 gravity gradients and cover an area of about 45 000 km^2 (See statistical parameters in Table 1). The covariance matrix thus has slightly more than 2 billion elements and that would require 15 GB capacity to store the full matrix in double precision.

To make our problem numerically tractable, we have two choices. First, we could use instead of (5) a compactly supported covariance function (Gneiting, 2002). Our second choice is to keep the covariance model (5) with infinite support, but neglect the covariances beyond a certain distance d_{max}. Through either of these achievements the covariance matrix will be sparse and thus efficient sparse matrix techniques can be used.

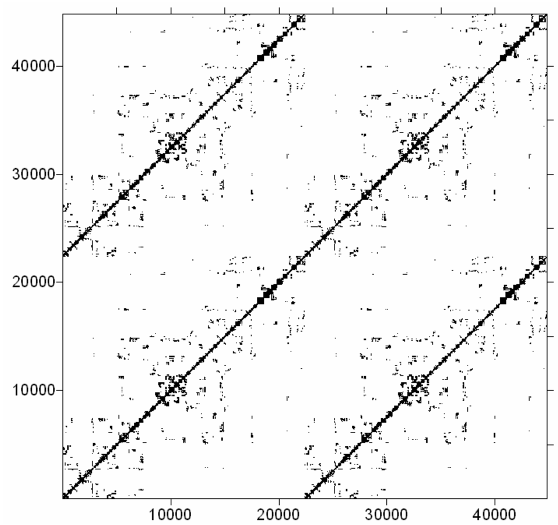

Fig. 2 Nonzero pattern of the sparse $C_{\ell\ell}$ matrix (4) before preordering. The number of nonzero elements is 4 049 268 or 0.2%.

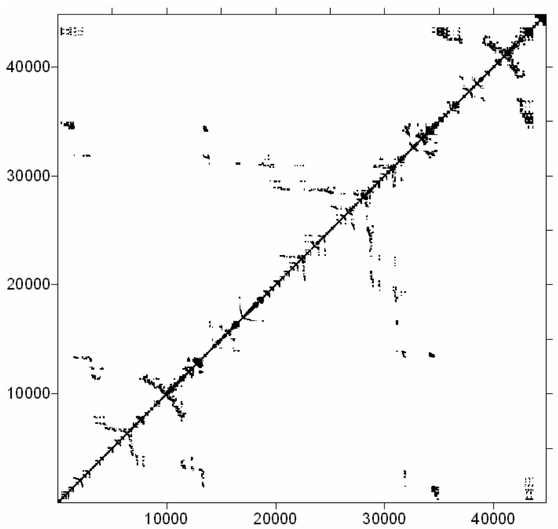

Fig. 3 Nonzero pattern of the $C_{\ell\ell}$ matrix (4) after approximate minimum degree (AMD) preordering

Our example computations were based on the second choice. All auto- and cross-covariances were truncated in the same way at d_{max}. If the parameters of the Gaussian covariance function are $A = 0.5$

mGal^2 and $d_0 = 1.5$ km, the covariance drops below 5% of its maximum value at $d_{max} = 4$ km. Beyond this maximum distance all covariances were considered to be zero. Truncating covariances like this will tend to remove from the estimated Δg field any long-range (wavelength d_{max}) systematic patterns present in the gradiometric data. This way the number of nonzero elements in the covariance matrix has been reduced to 4 049 268. In efficient compressed column format (Davies, 2005) with the necessary bookkeeping information the matrix can be stored in 46 MB. It was found that about 300 MB in-core memory was consumed during the assembly and solution stages of the problem, which is entirely acceptable even on a standard PC.

Table 1 Statistical parameters of horizontal gravity gradients used in the calculations. All units are E (1 Eötvös = 10^{-9} s^{-2})

	min	max	mean	std
V_{xz}	-82.80	98.90	-0.45	11.17
V_{yz}	-173.90	225.40	0.92	11.79

For numerical tests we have developed Fortan 90 code and interfaced it with the C language LDL library (Davies, 2005). The preordering of the matrix for efficient factorization was performed through calls to the AMD library (Amestroy et al., 2004). It can be seen that the original covariance matrix (Fig. 2) after the approximate minimum degree (AMD) preordering step has a number of large nonzero blocks (Fig. 3), and this permutation prevents fill-in during the next step, the Cholesky factorization of the matrix.

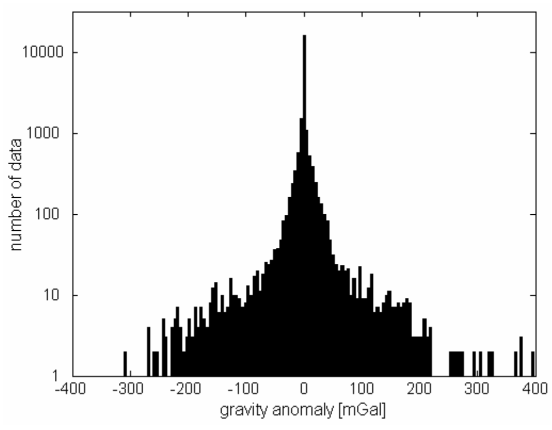

Fig. 4 Histogram of gravity anomalies predicted from horizontal gravity gradients at 22409 points with 150 bins. Notice the logarithmic scale on the vertical axis. Only less than 1% of the predictions are above ±25 mGal.

The solution of the sparse linear system (1) provided us gravity anomalies at 22 409 points. The measurements were considered to be uncorrelated and with uniform noise variance. After several tests runs the noise standard deviation of horizontal gravity gradients was chosen to be ±13.5 E. With this value the variance of predicted gravity anomalies was in agreement with the variance of the chosen covariance model. Moreover, this noise variance level is in agreement with the actual errors of ±10-15 mGal found by Hein (1981) from his collocation experiments with horizontal gravity gradients in the Upper Rhine Valley in Germany. The histogram of predicted gravity anomalies can be seen on Fig. 4. Although there are several extremely big values (up to 400 mGal!), these are restricted only to a small area and more than 99% of the predictions fall within the ±25 mGal range.

It was interesting to us to make comparisons of these results with gridded 1'×1.5' free-air gravity anomalies. These anomalies were reduced to the effect of the EGM96 geopotential model. To get comparison also with the high frequency part of gravity anomalies, low-pass filtered anomalies with a Gaussian filter of length 15 km were removed. We found that the agreement seems better with high-pass filtered gravity anomalies (Fig. 5) than with the original ones. This was expected, since our previous experiences have shown that gravity gradients are more sensitive to local features of the gravity field than gravity anomalies.

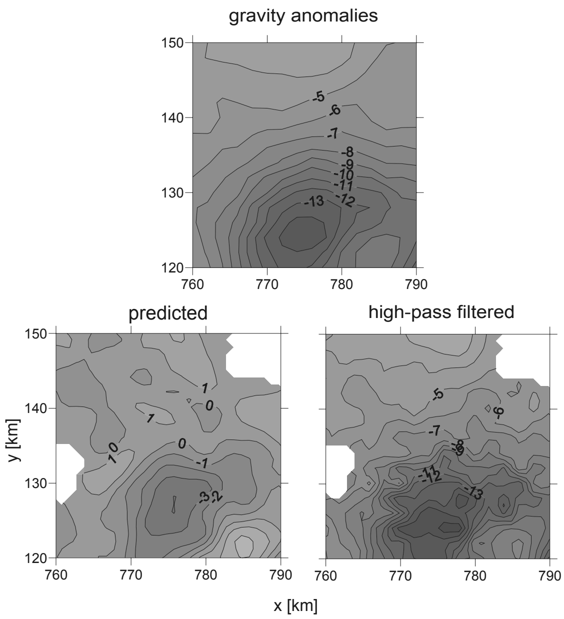

Fig. 5 Comparison of gravity anomalies, high-pass filtered gravity anomalies and predictions over a selected 30x30 km^2 nearly flat area. Contour interval is 1 mGal.

Fig. 5 also shows that predicted gravity anomalies in this region have less power than the actual gravity field. Truncating covariances would partially account for the loss of signal in these areas. On the other hand the chosen covariance model may be inappropriate for this almost flat area – especially the correlation length is too small. On the other hand if the area has non-flat topography, the predicted anomalies have considerably more power than reference gravity anomalies (Fig. 6). This raises the problem of non-stationarity of the gravity gradient signal, the variance of which is very strongly correlated with the topography of the area. The histogram of average point distances (Fig. 7) reflects the difference between flat and non-flat areas as well. This is a problem for least-squares collocation, since homogeneity and isotropy are essential assumptions of the method (Kearsley, 1977). Other methods like kriging may be interesting in this respect, which do not require stationarity assumption on the signal, only stationarity of signal increments, i.e. the intrinsical stationarity (Gneiting et al., 2000). We have also the possibility to smooth the gravity field by removing additional topographical effects from the gravity gradient signal or to make the predictions separately for flat and non-flat areas.

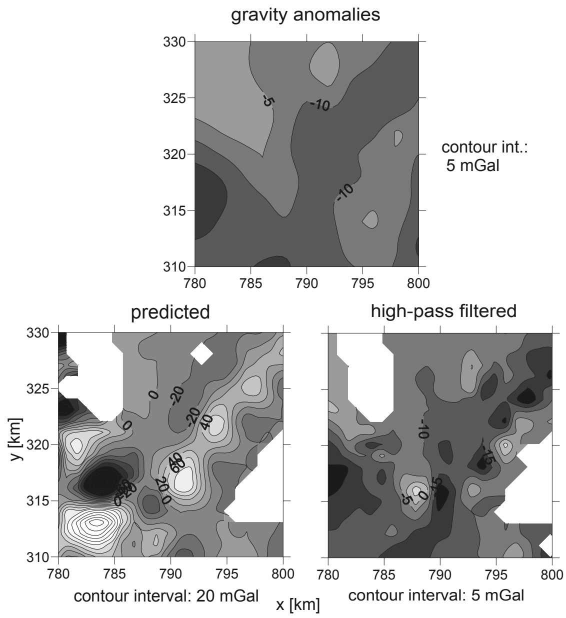

Fig. 6 Comparison of gravity anomalies, high-pass filtered gravity anomalies and predictions over a selected 20x20 km^2 non-flat area. Contour interval is 5 mGal for the upper and right subfigures and 20 mGal for left subfigure. Notice the very high variance of predicted gravity anomalies from the horizontal gradients

4 Conclusions and recommendations

In the present study it was shown how efficient sparse matrix techniques can be used in local gravity field modeling with horizontal gravity gradients. The example computation with Hungarian gravity gradient data has suggested that non-stationarity of gravity gradients makes it difficult to achieve a uniformly good prediction in areas of different topography. On the other hand gravity anomalies predicted from horizontal gradients may show significant details at short wavelengths of the gravity field which are not necessarily present in gravity anomalies.

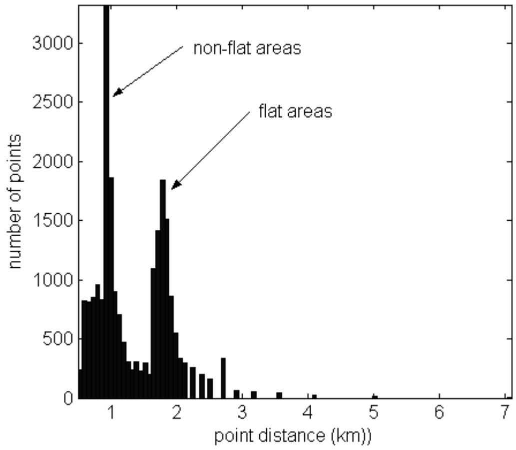

Fig. 7 Histogram of average point distances of gravity gradient observations. It can be observed that flat and non-flat areas have different average point distances

Hence we propose to use gravity gradients together with topography and gravity measurements to yield a better model of the local gravity field than from gravity measurements alone. Our results have shown, however, that the topography of the area has a strong impact on the gravity gradient signal and it must be considered carefully. Further tests should also be done with other covariance models and especially compactly supported covariance functions. Non-stationarity of gravity gradients can be a problem in combined modeling of the gravity field and it deserves further attention and research.

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